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OF

LINES, AREAS, SURFACES, AND VOLUMES,

COMBINING

ELEMENTS OF THE SCREW-PROPELLER AND NAVAL ARCHITECTURE GENERALLY,

WITH THE

APPLICATION OF ALGEBRA TO MENSURATION.

DESIGNED FOR THE USE OF SCHOOLS AND ARTIZANS.

BY

ROBERT RAWSON, ESQ.,

Head Master of the Dockyard School, Portsmouth; Honorary Member of the Manchester Literary and Philosophical Society.

LONDON:
WHITTAKER AND CO., AVE MARIA LANE.

1856.

LONDON:
PRINTED BY A. D. MILLS, CRANE-COURT, FLEET-STREET.

TO GEORGE VAUGHAN, ESQ.,

SIR,

I respectfully beg to inscribe to you the following pages, as a sincere mark of the estimation in which I hold your personal character and encouragement of science, and of gratitude for the kindly feelings you have invariably manifested towards me.

I have the honour to be,
Sir,
Your obedient humble servant,
ROBERT RAWSON.

Highland House, Portsea, Hants,
May 1st, 1856.

PREFACE.

It will be readily seen, by a cursory glance through the following pages, that the matter and arrangement differ considerably from those of the books on Mensuration now in use. By this statement I do not wish to imply either that the works of preceding writers are deficient in value, or that this production will render future labours in this branch of science unnecessary; as but a slight experience in teaching is sufficient to convince any one, possessing moderate views and reasonable discrimination, of the great difficulty in selecting and arranging matter of this kind to meet the requirements, views, tastes, and expectations of teachers and students in general.

As the deviation from the plan usually adopted by other writers is considerable, and, in my opinion, important, I feel that some explanation is necessary, in order that my views and objects may be intelligible to the reader. It would be in vain to deny that there are but few books on Mensuration, or indeed on any other subject, free from defects, both numerous and serious; and being fully convinced of this universal truth, it would be presumptuous, and wanting in judgment on my part, to hope that this book will be an exception to this unfortunate fatality.

I trust that necessity and experience have made me too well acquainted with my own insufficiency ever to entertain the realization of such a visionary expectation, but still I have been extremely anxious to supply, consistently with the time at my disposal, and to the utmost of my ability,

1. Matter and examples, directly applicable to Naval Architecture and Screw Propulsion;

2. An extensive number of examples in the more prominent rules *of Mensuration* with a view to afford the student additional *practice* in Arithmetic and Algebra.

Amongst the new subjects which have been introduced in this work may be mentioned, The finding of the metacentre; The displacement of various solids; The areas and centres of gravity of curves by Simpson's rule; The displacement and centre of gravity of a ship, together with its statical and dynamical stability; The length of the helix of a screw-propeller, with its area and angle, by means of a table which will facilitate considerably the computation of these elements. I trust that the whole of this additional matter of increasing interest and importance will be found useful to those who are engaged in the construction and management of the great bulwarks of our country.

The examples, which, in the ordinary rules, are numerous, have been considered with the object of eliciting several answers from the same data, a course which will be best illustrated by means of a quotation, viz., "Find the surface, volume, weight, and displacement of a globe of English oak whose diameter is 24 feet 7 inches." The reasons for such a procedure must be obvious to all engaged in the arduous and important office of teaching. With a view to impress the algebraical formulæ, which state the rules, more strongly upon the memory, and at the same time to afford considerable practice in algebraical equations, there is given a great number of inverse questions and miscellaneous examples, which require for their solution some acquaintance with the principles and practice of Algebra. In connection with my own experience I have found the working of these inverse questions attended with advantage, both as an exercise in the use of algebraical symbols, and in the solution of equations which commonly occur in the consideration of practical questions. It has been deemed advisable to follow, generally, the usual plan of stating the rules in ordinary language as well as by algebraical formulæ. I hope, however, as the advantages of education become more extensive and better appreciated by the artisans of this country, that the former mode will be entirely abolished and yield its position to the supremacy of the latter, which is far more intelligible and effective. An acquaintance with the ordinary rules of addition, subtraction, multiplication, and division of Algebra, together with the simplest axioms of Geometry, a task by no means difficult to be accomplished even by the humblest artisan, will readily enable a person to multiply his knowledge manifold with certainty and success. In order to illustrate what I mean I [redacted]use of the following relation :—

$$W = \frac{\pi \delta D^3}{6}$$

which expresses the weight (W) of a sphere whose diameter is D and density δ. Suppose, in the first place, that the weight and density of the sphere were known quantities and the diameter was required. In this case, both sides of the above equation must be multiplied by 6 and divided by $\pi \delta$, and the cube root of the result extracted; then the formula will stand thus:—

$$D = \left\{ \frac{6\ W}{\pi\ \delta} \right\}^{\frac{1}{3}}, \text{ which gives the diameter.}$$

Suppose, in the second place, that the weight and diameter of the sphere were known quantities, and the density was required.

Then, both sides of the first equation must be multiplied by 6, and divided by the product of π and D^3, and the equation will become

$$\delta = \frac{6\ W}{\pi\ D^3}, \text{ which gives the density.}$$

(The density of a material body is the weight of a cubic foot of it.)

The two latter problems are inverse problems, and require for their solution the application of the simplest axioms and the easiest principles. The demonstrations of the rules by the application of geometrical and algebraical formulæ have been, for the most part, omitted, in accordance with the plan which, after considerable experience, has been thought advisable to follow. In the mensuration of lines, areas, and volumes or solids, it is customary for mathematicians to conceive a line to be generated by the motion of a point; an area to be produced by the motion of a line; and a solid or volume to be the result of the motion of a plane; and in order that this motion may be the interpretation of known curved lines, areas, and volumes of various forms, it is necessary to suppose a point in motion to be constantly changing its direction, a line in motion to be constantly changing its length and direction, and a plane in motion to be constantly changing its area and direction.

From this view of the subject it will be obvious, after a little reflection, that, with few exceptions, the artifices and resources of the higher mathematics (particularly the integral calculus) are absolutely necessary to enable any one to investigate

success, the rules and formulæ for the length of curves, areas, and volumes. Feeling the force of this consideration, I could not resist the conviction that the demonstrations of the rules in a work of this kind would be useless to those who are familiar with the principles and processes of the integral calculus, and could not be given in a way to be useful to those unacquainted with the science. With respect to the history of this portion of practical geometry, it would be difficult to record, with accuracy and fidelity, the important labours of each mathematician, whose genius and investigations have contributed so largely to its present advanced state of usefulness; still, the particular history of the famous ancient problem, "*Squaring the Circle*," is so interesting and well defined as to justify its insertion in a work of this nature. The *squaring of a circle* is a problem, the object of which is to construct, geometrically, with a rule and compass, a square equal in area to a given circle. The artifices, modes of thought, and profound geometrical skill, which have been employed by the humblest as well as by the most successful cultivators of pure science, in every age and in every civilised country, since the time of Archimedes, have not accomplished the object proposed; but have simply pointed out, to those who can seize the force of their developments, the impossibility of the problem. It may be stated that the "*Squaring of the Circle*" is readily reduced to the problem, "*to find the ratio of the circumference of a circle to its diameter*," and it is this enquiry which has received the greatest share of attention. In consequence of the frequent application of the ratio here alluded to, in various branches of mathematics, it is usual to designate it by the symbol π.

Archimedes, the prince of ancient philosophers, was the first mathematician whose labours in connection with this problem were crowned with practical success. He showed that the value of π was nearly equal to $\frac{22}{7} = 3{\cdot}1428571$. This approximation being a little too great is very near the truth, and, in consequence of its simplicity, is still used by many practical men.

It was discovered by means of a circumscribed regular polygon of 192 sides, and an inscribed regular polygon of 96 sides.

Vieta and Metius found the value of π to be $\frac{355}{113} = 3{\cdot}1415929$, *which is more correct than* the ratio of Archimedes, but not so

ple.

By the continual bisection of an arc of a circle, Van Ceulen, a Dutchman, determined the ratio π to be 3·1415926535, &c., to 36 places of decimals. This result was obtained by troublesome and laborious calculations, and was thought so curious that the numbers were cut on his tombstone in St. Peter's Churchyard, at Leyden.

Abraham Sharp, of Little Horton, near Bradford, Yorkshire, extended the approximation of Van Ceulen to 72 places of decimals.

The discovery of the differential and integral calculus, the summation of infinite series, and the powerful notation of trigonometrical functions, have enabled modern mathematicians to devise series and formulæ by which the value of π can be more readily approximated to than by the inscribed and circumscribed regular polygons of the ancients.

Machin, Professor of Astronomy in Gresham College, extended the ratio π to 100 places of decimals by meams of the formula,

$$\frac{\pi}{4} = 4\tan.^{-1}\left(\frac{1}{5}\right) - \tan.^{-1}\left(\frac{1}{239}\right)$$

Before 1831, by the labours of De Lagny, Euler, and other celebrated mathematicians, the value of π had been calculated to the extent of 140 places of decimals.

In 1841, Dr. Rutherford, Royal Military Academy, Woolwich, calculated the value of π to 208 decimals, by the formula

$$\frac{\pi}{4} = 4\tan.^{-1}\left(\frac{1}{5}\right) - \tan.^{-1}\left(\frac{1}{70}\right) + \tan.^{-1}\left(\frac{1}{99}\right)$$

In 1846, M. Dase found the value of π to 200 decimals from the formula

$$\frac{\pi}{4} = \tan.^{-1}\left(\frac{1}{2}\right) + \tan.^{-1}\left(\frac{1}{5}\right) + \tan.^{-1}\left(\frac{1}{8}\right)$$

In 1847, Dr. Clausen, of Dorpat, pushed the approximation of π to 250 decimals by means of the formulæ

$$\frac{\pi}{4} = 4\tan.^{-1}\left(\frac{1}{5}\right) - \tan.^{-1}\left(\frac{1}{239}\right)$$

$$\text{And, } \frac{\pi}{4} = 2\tan.^{-1}\left(\frac{1}{3}\right) + \tan.^{-1}\left(\frac{1}{7}\right)$$

(See "Rectification of the Circle to 607 Places of Decimals," by Mr. Shanks.)

In 1851, William Shanks, Houghton-le-Spring, Durham, calculated, by means of Machin's formula, the value of π to 315 decimals. This gentleman and Dr. Rutherford have since pushed the approximation of π to the extent of 441 decimals. The computations were independent of each other, and therefore the decimals to this extent may be pronounced free from errors.

With great industry, Mr. Shanks has pushed the approximation still further, viz., to 607 places of decimals; for the accuracy of the last 166 figures he is alone responsible.

The value of π, as calculated by Mr. Shanks, is

3·1415926 5358979 3238462 6433832 7950288 4197169
3993751 0582097 4944592 3078164 0628620 8998628
0348253 4211706 7982148 0865132 8230664 7093844
6095505 8223172 5359408 1284811 1745028 4102701
9385211 0555964 4622948 9549303 8196442 8810975
6659334 4612847 5648233 7867831 6527120 1909145
6485669 2346034 8610454 3266482 1339360 7260249
1412737 2458700 6606315 5881748 8152092 0962829
2540917 1536436 7892590 3600113 3053054 8820466
5213841 4695194 1511609 4330572 7036575 9591953
0921861 1738193 2611793 1051185 4807446 2379834
7495673 5188575 2724891 2279381 8301194 9129833
6733624 4193664 3086021 3950160 9244807 7230943
6995530 9662027 5569397 9869502 2247499 6206074
9703041 2366929 13332 + &c.

The decimals have been computed by Machin's formula

$$\frac{\pi}{4} = \tan.^{-1}\left(\frac{1}{5}\right) - \tan.^{-1}\left(\frac{1}{239}\right)$$

with the assistance of the well-known series

$$\tan.^{-1}\left(\frac{1}{5}\right) = \frac{1}{5} - \frac{1}{3\cdot 5^3} + \frac{1}{5\cdot 5^5} - \frac{1}{7\cdot 5^7}$$

$$+ \frac{1}{9\cdot 5^9} - \frac{1}{11\cdot 5^{11}} + \&c.$$

$$\text{And, } \tan.^{-1}\left(\frac{1}{239}\right) = \frac{1}{239} - \frac{1}{3\cdot 239^3} + \frac{1}{5\cdot 239^5}$$

$$- \frac{1}{7\cdot 239^7} + \frac{1}{9\cdot 239^9} - \text{\&c.}$$

Those who are anxious for further information respecting the history of this ancient problem may be referred to the "Penny Cyclopædia," by Professor De Morgan, and the Rectification of the Circle, by Mr. Shanks.

The latter has computed $\tan.^{-1}\left(\frac{1}{5}\right)$ and $\tan.^{-1}\left(\frac{1}{239}\right)$ each to 609 decimals, the base of Napier's logarithms to 137 decimals, the modulus of the common system of logarithms to 136 places of decimals, and the powers of 2 as far as 2^{721}, commencing with 2^{18}, and proceeding 2^{25}, 2^{37}, 2^{49}, 2^{61}, &c.

I shall now conclude with the hope that what I have done may be useful to the teacher, the pupil, and the artisan, in facilitating the study and practice of a pleasing and important science, which has numerous applications in the ordinary course of practical experience.

ROBERT RAWSON.

PORTSMOUTH, *May* 1*st*, 1856.

EUCLID'S DEFINITIONS.

I.

A point is that which hath no parts, or which hath no magnitude.

II.

A line is length without breadth.

III.

The extremities of a line are points.

IV.

A straight line is that which lies evenly between its extreme points.

V.

A superficies is that which hath only length and breadth.

VI.

The extremities of a superficies are lines.

VII.

A plane superficies is that in which any two points being taken, the straight line between them lies wholly in that superficies.

VIII.

"A plane angle is the inclination of two lines to one another in a plane, which meet together but are not in the same direction."

IX.

A plane rectilineal angle is the inclination of two straight lines to one another, which meet together, but are not in the same straight line.

N.B.—"*When several* angles are at one point B, any one of *them is expressed by* three letters, of which the letter that is at *vertex of the angle, that is,* at the point in which the straight

lines that contain the angle meet one another, is put between the other two letters, and one of these two is somewhere upon one of those straight lines, and the other upon the other line: Thus the angle which is contained by the straight lines, *A B*, *C B*, is named the angle *A B C*, or *C B A*; that which is contained by *A B*, *D B*, is named the angle *A B D*, or *D B A*; and that which is contained by *D B*, *C B*, is called the angle *D B C*, or *C B D*; but, if there be only one angle at a point, it may be expressed by a letter placed at that point; as the angle at *E*."

X.

When a straight line standing on another straight line makes the adjacent angles equal to one another, each of the angles is called a right angle; and the straight line which stands on the other is called a perpendicular to it.

XI.

An obtuse angle is that which is greater than a right angle.

XII.

An acute angle is that which is less than a right angle.

XIII.

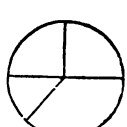
"A term or boundary is the extremity of any thing."

XIV.

A figure is that which is inclosed by one or more boundaries.

XV.

A circle is a plane figure contained by one line, which is called the circumference, and is such that all straight lines drawn from a certain point within the figure to the circumference, are equal to one another.



XVI.

And this point is called the centre of the circle.

XVII.

A diameter of a circle is a straight line drawn through the centre, and terminated both ways by the circumference.

XVIII.

A semicircle is the figure contained by a diameter and the part of the circumference cut off by the diameter.

XIX.

"A segment of a circle is the figure contained by a straight line, and the circumference it cuts off."

XX.

Rectilineal figures are those which are contained by straight lines.

XXI.

Trilateral figures, or triangles, by three straight lines.

XXII.

Quadrilateral, by four straight lines.

XXIII.

Multilateral figures, or polygons, by more than four straight lines.

XXIV.

Of three-sided figures, an equilateral triangle is that which has three equal sides.

XXV.

An isosceles triangle is that which has only two sides equal.

XXVI.

A scalene triangle is that which has three unequal sides.

XXVII.

A right-angled triangle is that which has a right angle.

XXVIII.

An obtuse-angled triangle is that which has an obtuse angle.

XXIX.

An acute-angled triangle is that which has three acute *angles.*

XXX.

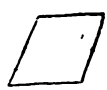
Of four-sided figures, a square is that which has all its sides equal, and all its angles right angles.

XXXI.

An oblong is that which has all its angles right angles, but has not all its sides equal.

XXXII.

A rhombus is that which has all its sides equal, but its angles are not right angles.



XXXIII.

A rhomboid is that which has its opposite sides equal to one another, but all its sides are not equal, nor its angles right angles.

XXXIV.

All other four-sided figures besides these, are called Trapeziums.

XXXV.

Parallel straight lines are such as are in the same plane, and which being produced ever so far both ways, do not meet.

POSTULATES.

I.

Let it be granted, that a straight line may be drawn from any one point to any other point.

II.

That a terminated straight line may be produced to any length in a straight line.

III.

And that a circle may be described from any centre, at any distance from that centre.

AXIOMS.

I.

Things which are equal to the same thing, are equal to one another.

II.

If equals be added to equals, the wholes are equal.

III.

If equals be taken from equals, the remainders are equal.

IV.

If equals be added to unequals, the wholes are unequal.

V.

If equals be taken from unequals, the remainders are unequal.

VI.

Things which are double of the same, are equal to one another.

VII.

Things which are halves of the same, are equal to one another.

VIII.

Magnitudes which coincide with one another, that is, which exactly fill the same space, are equal to one another.

IX.

The whole is greater than its part.

X.

Two straight lines cannot inclose a space.

XI.

All right angles are equal to one another.

XII.

"If a straight line meets two straight lines, so as to make the two interior angles on the same side of it taken together less than two right angles, these straight lines being continually produced, *shall at length meet* upon that side on which are the *angles which are less than* two right angles."

EXPLANATION OF TERMS AND USEFUL NUMBERS IN ARITHMETICAL CALCULATION.

In accordance with the design of this book, terms have been introduced which are not in ordinary use in works of this kind, and which require a little explanation. I trust the following will be deemed sufficient to answer every purpose :—

Pitch of a Screw.—It is the vertical distance between two threads of a screw, or rather the distance between two consecutive threads. A tolerable idea of a screw may be formed by cutting an inclined plane out of paper and wrapping it round a cylinder whose length is equal to the height of the plane, and whose circumference of base is equal to the base of the inclined plane. The height of the inclined plane is the *pitch* of the screw.

Angle and helix of screw.—The angle at the base of the inclined plane is the *angle* of the screw, and it is the same as the angle *D B C* in diagram, page 24. The hypothenuse of the inclined plane is the *helix* of the screw, and corresponds with the line *B D*, in diagram, page 24.

Screws in general use are formed by numerous convolutions of the inclined plane, or by repeating the process of wrapping the inclined plane round the cylinder, increased in length any convenient number of times. In this manner the screw has a *length*, which is equal to the vertical distance between the extremities of the *helix*. The length of the screws, however, employed in propelling vessels is usually about one-sixth of their pitch, and therefore the helix does not form a complete convolution. The pitch of such screws is an abstract magnitude, and can only be measured by means of its relation to the diameter, length, and angle of the screw. The screw propeller-blade, properly speaking, is a portion of a conoidal surface, which is produced by a straight line *A* (always perpendicular to *B*), moving uniformly round *B*, while its extremity moves uniformly in the direction of B.

Positive and negative slip of screw.—The slip of a screw is explained in Problem 9, page 25. When the vessel is moving slow

than the screw, then the slip is *positive*, and it is *negative* when the vessel is moving faster than the screw. The fact of the vessel going faster than the screw which propels it, is deemed by many an anomaly in screw propulsion, and there are those who believe that such a circumstance is impossible, and contrary to the received laws of mechanics. I cannot coincide with this view, as there are several well authenticated facts which prove the contrary, and the reason for the negative slip is not difficult to assign when it is remembered, that the screw revolves in water at the stern of the vessel which moves in the direction of the vessel's motion. When screw propulsion was first introduced, the slip of the screw claimed considerable attention, and it was thought by many that a diminution of this element would be attended by an increase in the speed of the vessel. Hence, the determination of the pitch, diameter, and length of a screw, so as to produce a minimum slip, was a great desideratum. Experience, however, was not slow in correcting this hasty impression. Screws were obtained which produced little or no slip, still the speed of the vessel was not improved: the reason of this result is by no means difficult to understand, as the slip depends on two elements, viz., the speed of the vessel and the rotatory motion of the screw.

Metacentre.—This term was first introduced by Bougier, and is the ultimate intersection of a perpendicular to the curve, which is the locus of the centre of buoyancy, with the line passing through the centre of gravity of the ship and the centre of gravity of the part immersed when the ship is not inclined.

Displacement of a ship is the quantity of water, in cubic feet, or in tons, displaced by the ship when it floats.

Solids and Surfaces of Revolution.

A sphere is formed by the revolution of a semicircle about its diameter.

A cone is produced by the revolution of a right-angled triangle about one of the sides which contains the right angle.

A paraboloid is generated by the revolution of a parabola about its abscissa. See Problem 20, page 7.

Prolate spheroid is produced by the revolution of an ellipse about its transverse, or longest diameter.

Oblate spheroid is produced by the revolution of an ellipse about its conjugate, or shortest diameter.

This is the figure which the earth would assume, supposing it to be originally composed of a homogeneous fluid mass, every particle attracting every other particle with an accelerating force, proportional to the mass of the attracting particle directly, and the square of the distance of the attracted particle inversely. The whole fluid mass revolving about an axis in 23 h. 56 m. 4 s. (See Airy's Tracts, page 136.)

Hyperboloid is formed by the revolution of a hyperbola about its transverse axis.

Ellipsoid is a solid bounded by a surface, the equation of which is $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, the constants a, b, c are the semi-diameters, and x, y, z the distances of a point on the surface from the centre, measured in the direction of the diameters respectively. This surface may be generated by a variable ellipse moving upwards parallel to itself, with its centre on a line perpendicular to the plane of the ellipse.

Useful numbers in arithmetical calculations.

$\pi = 3{\cdot}1415926536.$

$\frac{1}{\pi} = {\cdot}3183098862.$

Log. $\pi = {\cdot}4971498727.$

$\sqrt{\pi} = 1{\cdot}7724538509.$

$\sqrt{\frac{1}{\pi}} = {\cdot}5641895835.$

$\pi^2 = 9{\cdot}8696044011.$

$\frac{1}{\pi^2} = {\cdot}1013211836.$

$\epsilon = 2{\cdot}7182818284$ — Base of Napierian System of Logarithms.

$M = {\cdot}4342944819$ — Modulus of common Logarithms.

$\sqrt{2} = 1{\cdot}4142135624.$

$\frac{1}{\sqrt{2}} = {\cdot}7071067812.$

$\pi\sqrt{2} = 4{\cdot}4428829381.$

$\pi\sqrt{\frac{1}{2}} = 2{\cdot}2214414691.$

$\sqrt{\frac{\pi}{2}} = 1{\cdot}2533141373.$

$\sqrt{\frac{2}{\pi}} = {\cdot}7978845608.$

$\frac{\sqrt{2}}{\pi} = {\cdot}4501581581.$

$g = 32{\cdot}19$ — accelerating force of gravity.

$\sqrt{g} = 5{\cdot}67363.$

Cubic inch of distilled water, Baro. 30 ins.,
Ther. 62° . . . . . = 252·458 grs.
Cubic inches in one ounce of water . . = 1·73298.
Cubic inches in the imperial gallon . . = 277·276.
Length of second's pendulum at London
(Bessel) . . . . . = 39·13907 ins.
Cubic foot of air, Fah. 62° . . . = 528·367 grs.
Cubic feet in a ton of granite . . . = 13·505.
" " marble . . . = 13·070.
" " chalk . . . = 12·874.
" " limestone . . = 11·273.
" " elm. . . . = 64·460.
" " Honduras mahogany = 64.
" " Mar Forest fir . = 51·65.
" " beech . . . = 51·494.
" " Riga fir . . . = 47·762.
" " Ash and Dantzic oak = 47·158.
" " Spanish mahogany. = 42·066.
" " English oak . . = 36·205.
" " Sea-water . . = 34·902.
" " Rain-water . . = 34·24.

(From Moseley's Engineering and Architecture.)

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9	3 from bottom	*A B*	*A P*.
17	Question 7.	16·9	16 feet 9 inches.
31	Sec. 1, Ques. 1.	and its base	and its perpendicular.
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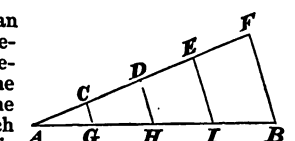
4	Sec. 1, Ques. 1.	5770	5670.
8	Sec. 1, Ques. 1.	140	104.

PART THE FIRST.

PRACTICAL GEOMETRY.

Problem 1.

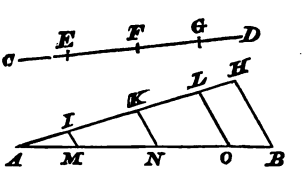
To divide a line A B into any number of equal parts.



Draw *A F*, in any direction, to an unlimited length, take any convenient arbitrary length *A C*, and repeat it along the line *A F*, at the points *C*, *D*, &c., as often as the number of equal parts into which the line *A B* is proposed to be divided. In this case it is four; join *F B*, parallel to which draw *E I*, *D H*, *C G*, then the line *A B* is divided in the points *G*, *H*, *I*.

Problem 2.

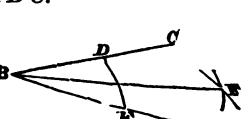
To divide a line A B in the same proportion as a given line C D.



Draw *A H*, in any direction, equal to *C D*, and place upon it *A I*, *I K*, *K L*, &c., equal to *C E*, *E F*, *F G*, &c.; join *H B*, parallel to which draw *L O*, *K N*, *I M*; then *A B* is divided in the points *M*, *N*, *O*, in the same proportion as *C D*.

Problem 3.

To bisect a given angle, A B C.

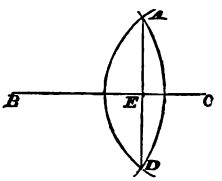


With the centre, *B*, and any radius, *B E*, describe the arc *D E*; and with the same radius, or less if more convenient, *and centres D* and *E*, describe *arcs cutting each* other in *F*, join *F B*; *then F B bisects* the angle *A B C*.

Problem 4.

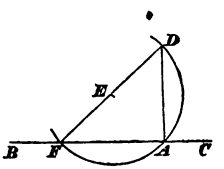
From a given point A, to draw a line perpendicular to B C.

With centre *B* and radius *B A* describe the arc *A D*, with the centre *C* and radius *C A* describe the arc *A D*, join *A D* cutting *B C* in *E*; then *E A* is the perpendicular required.



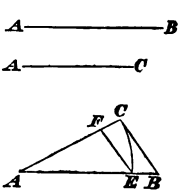
If the point *A* be in the line *B C* the above construction fails.

Take any point, *E*, and with the radius *E A* describe the circle *D A F*, join *F E* and produce it to meet the circle in *D*; join *D A*, which is the perpendicular required.



Problem 5.

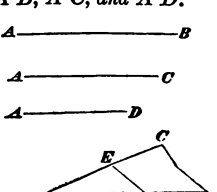
To find a third proportional to the lines A B and A C.



Place *A B* and *A C* making an angle at *A*, join *C B*, and with the centre *A* and radius *A C* describe the circle *C E*, draw *E F* parallel to *B C*; cutting *A C* in *F*, then *A F* is the third proportional required.

Problem 6.

To find the fourth proportional to the lines A B, A C, and A D.

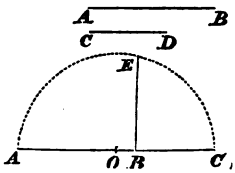


Place *A B*, *A C*, making an angle at *A*, join *C B* and draw *D E* parallel to *B C*; then *A E* is the fourth proportional required.

Problem 7.

To find a mean proportional to two lines, A B, C D.

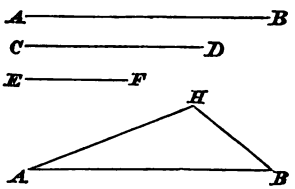
Place *A B*, *C D* in the same straight line and bisect it in *O*, with the centre *O* and radius *O C* describe the semicircle *A E C*, draw *B E* perpendicular to *A C*, then *B E* is the mean proportional required.



Problem 8.

To make a triangle with the given straight lines A B, C D, E F.

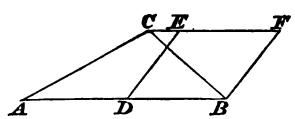
Take *A B* as the base, and with the centre *A* and radius *C D* describe the arc at *H*, with the centre *B* and radius *E F* describe the arc at *H*, join *A H*, *B H*, and the triangle *A B H* is the triangle required.



Problem 9.

To make a parallelogram equal to a triangle A B C.

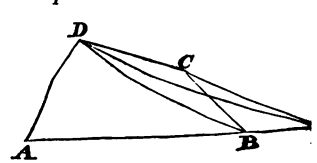
Bisect *A B* in *D*, draw *D E* making any given angle with *A B*, draw *C E F* parallel to *A B*, and *B F* parallel to *D E*; then *D E F B* is the parallelogram required.



Problem 10.

To make a triangle equal to the trapezium A B C D.

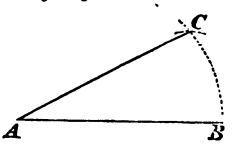
Produce *A B* to *E*, join *D B*, and draw *C E* paralled to *D B*, join *D E*, and the triangle *A D E* is the triangle required.



Problem 11.

To make an angle of any number of degrees.

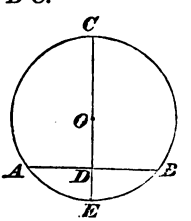
From a scale of chords take *A B* the chord of 60 degrees, and with the centre *A*, and radius *A B* describe the circle *B C*; with the centre *B*, and a radius equal to the chord of the given number of degrees describe the arc at *C*, join *A C*, and the angle *B A C* will be the angle required.



Problem 12.

To find the centre of the circle A B C.

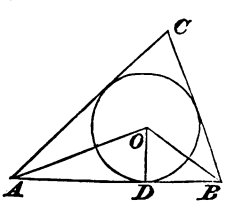
Take any chord, *A B*, and bisect it in *D*; draw *C D E* perpendicular to *A B*, and bisect it in *O*, which is the centre required.



Problem 13.

To inscribe a circle in a triangle A B C.

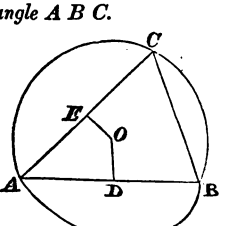
Bisect the angles *A B C*, *B A C*, by the lines *B O*, *A O* meeting in *O*. From *O* draw the perpendicular *O D*, then *O* is the centre, and *O D* is the radius of the inscribed circle.



Problem 14.

To describe a circle about a triangle A B C.

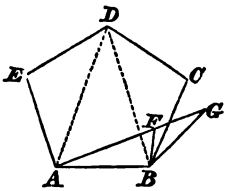
Bisect the sides *A B*, *A C* in the points *D* and *E*; draw *D O*, *E O* perpendicular to *A B*, and *A C* respectively, meeting in *O*. With centre *O* and radius *O A* describe the circle *A C B*.



Problem 15.

To make a pentagon on the line A B.

From *B* draw *B F* perpendicular to *A B*, and equal to half of it, join *A F* and produce it to *G*, making *F G* equal to *B F*. With centres *A* and *B* and radius *A G*, describe arcs cutting each other at *D*; with centres *A* and *D*, and radius *A B* describe arcs cutting each other at *E*; and with centres *B* and *D*, describe arcs cutting at *C*. Join *A E*, *D E*, *B C*, *D C*, which forms the pentagon.

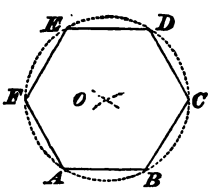


The line *B G* is the radius of the circle circumscribing the pentagon.

Problem 16.

To make a hexagon on a line A B.

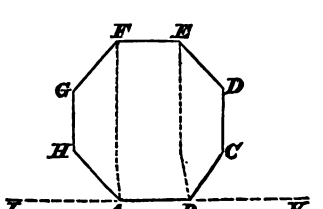
With the radius *A B* and centres *A* and *B*, describe arcs cutting in *O*. Then with the centre *O* and radius *A B*, describe the circle *A B C D E F*, and apply *A B* the necessary number of times round the circle, and the figure will be the hexagon.



Problem 17.

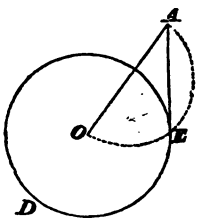
To make an octagon in a line A B.

Produce *A B* to *I* and *K*, and draw *A F*, *B E* perpendicular to *A B*. Bisect the angles *I A F*, *K B E*, and make *A H* and *B C* each equal to *A B*; draw *H G*, *C D* parallel to *A F*, and each equal to *A B*. With centres *G* and *D* and radius *A B* describe arcs cutting each other in *F* and *E*; join *G F*, *F E*, *E D*, and the octagon is complete.



Problem 18.

From a point A draw a line touching the circle E D.

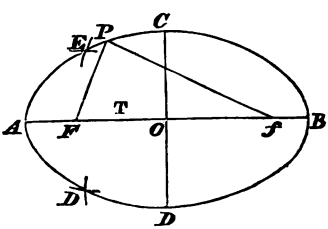


Find the centre *O* of the circle, join *A O*, upon which describe the semicircle, cutting the given circle in *E*, join *A E* which is the line touching the circle.

If the point *A* be on the circumference, this construction fails. In this case, join *O A*, to which draw a perpendicular from *A*; this will be the touching line required.

Problem 19.

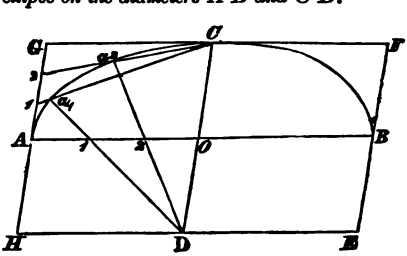
To describe an ellipse upon the diameters A B, C D, and centre O. The diameters being at right angles.



With centre *C* and radius *A O* describe arcs cutting *A B* in *F* and *f*; these points are called the foci of the ellipse. Take a thread equal in length to *A B* and fasten it with two pins at *F* and *f*; keep the thread tight, and it will reach the ellipse in *P*; and by moving a pencil round with the thread, observing always to keep it tight, the pencil will trace out the ellipse.

Otherwise.—Assume any point *T* on the transverse diameter *A B*, and with a radius *T A* and centre *F*, describe arcs at *D* and *E*, and with a radius *T B* and centre *f*, describe arcs at *D* and *E*; then the points *D* and *E* are in the curve of the ellipse.

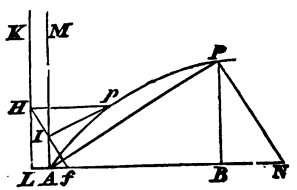
To describe an ellipse on the diameters A B and C D.



Draw the parallelogram *H F*; divide *A G* into any number of equal parts at 1, 2, &c., and also *A O* into the same number of equal parts at 1, 2, &c. Draw *C* 1 and *D* 1 meeting at a_1, and C 2, *D* 2 meeting at a_2, &c., then a_1 *and* a_2 *are points in the curve of the* ellipse. Proceed in the same manner for *each of the remaining* three quadrants.

Problem 20.

To describe a parabola with the abscissa A B, and ordinate B P, at right angles to it.



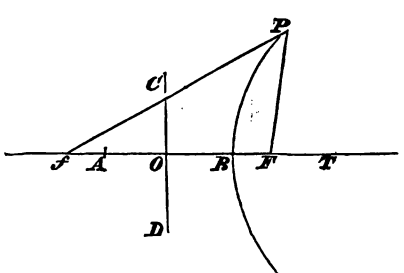
Join A P, and draw P N perpendicular to it, meeting A B produced in N; then B N is the parameter, one-fourth of which is the focal distance $A f$.

Make $A L = A f$, and draw L K and A M perpendicular to A B; in L K, the directrix, take any point H, and join f H cutting A M in I. Then, draw H p parallel to A B, and I p perpendicular to $H f$, and p is a point in the parabola.

Problem 21.

To describe an hyperbola on the diameters A B and C D at right angles to each other.

Make O F, $O f$ each equal to B C, then F and f are the foci of the hyperbola whose diameters are A B and C D. Produce A B indefinitely, and assume in it any point T; with T A as radius and centre f, describe an arc at P; with T B as radius, and centre F, describe an arc at P; then P is a point in the hyperbola.

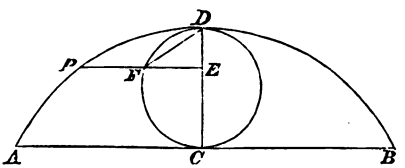


If the diameters be equal, the hyperbola is called equilateral. And if a parallelogram C O B N be drawn, the line O N produced is called an asymptote, which has the singular property constantly approaching the curve without ever meeting it.

Problem 22.

To describe a cycloid on the line A B.

Bisect *A B* in *C*, draw *C D* perpendicular to *A B* and equal to the diameter of a circle whose circumference is *A B*. In *C D* assume any point *E*, and draw *E F P* at right angles to *C D*, and make *P F* equal to the arc *D F* of the circle; then *P* is a point in the cycloid.



This curve was incidentally referred to by Aristotle, in alluding to the paradox, that a nail on the rim of a wheel describes a longer line than the axle. Roberval, Fermat, and Descartes shewed that a tangent at *P* is parallel to the chord *F D*, and the area *A D B* is equal to $\frac{3}{4}$ of *A B* × *C D*.

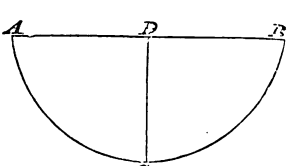
Huygens showed that the arc *P D* of the cycloid was double the chord *D F*; and also that a body would fall along it from any point, *P* to *D*, in the same time.

Newton and the Bernoullis showed it to be the curve of quickest descent, or that a body would fall by gravity from *A* to *D* in less time than by any other route.

Problem 23.

To describe a catenary.

A catenary is the curve which a regular and flexible chain or rope *A C B* will assume, if suspended from the ends *A* and *B*. Galileo confounded this curve with the parabola. Its correct nature was first detected by James Bernoulli. The equation to this curve is very difficult, and there is no easy method of describing it. Its application, however, to suspension bridges has given a considerable degree of importance to its various properties.



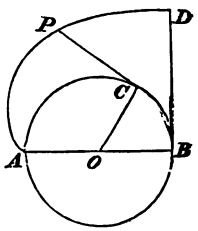
For further information, see Leslie's "Geometry of Curved *Lines," page 383.*

Problem 24.

To describe an involute of a circle A B C.

Draw the diameter *A B*, and in the circumference *A C B* take any point *C*. Join *O C*, and draw *P C* perpendicular to *O C*, or a tangent to the circle, making *P C* equal to the circumference *A C*, then *P* is a point in the involute.

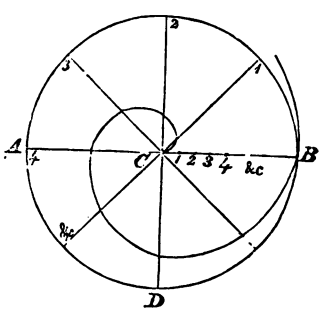
In toothed wheels there is less waste of power in passing from one tooth to another when they are of the form of the involute of a circle than in any other case.



Problem 25.

To describe the equable spiral of Archimedes.

With *C B* as a radius, describe the circle *A B D*. Divide the circumference into any number of equal parts 1, 2, 3, 4, &c., and the radius *C B* into the same number of equal parts 1, 2, 3, 4, &c., Join 1 *C*, 2 *C*, 3 *C*, 4 *C*, &c., on which take the distances 1 *C*, 2 *C*, 3 *C*, 4 *C*, &c., as given on *C B*, and these points will form the equable spiral of Archimedes.

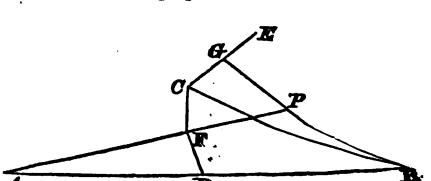


This curve is much used in architectural ornaments, and also for the volutes of the capitals of columns.

Problem 26.

To find any number of points in the circle passing through A B and C, where A D is equal to B D and C D perpendicular to A B.

Make the angle *E C B* equal to *D C B*, and in *C D* take any point *F*, through which draw *A P*. In *C E* take *C G* equal to *C F*, and draw *B G*, cutting *A B* in *P*; *then P is a point* in the *circle.*



MENSURATION OF LINES.

TABLE OF LINEAL MEASURE.

12 inches = 1 foot.
3 feet = 1 yard.
5½ yards = 1 pole.
40 poles = 1 furlong.
8 furlongs = 1 mile.

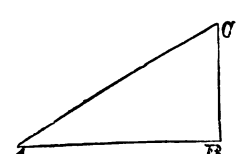
1 link = $7\frac{92}{100}$ inches = 7·92 inches.
1 chain = 100 links = 4 poles = 22 yards.
1 degree = 69½ English miles = 60 geographical miles.

Problem 1.

Any two sides of a right angled triangle being given, to find its third side.

1st. The lines *A B*, *B C* being given, to find *A C*.

Rule.—Square each of the sides, and add them together; the square root of this sum will be the side *A C*.



2nd. The lines *A C*, *A B* being given, to find *B C*.

Rule.—Square the lines *A C* and *A B*, and take their difference; the square root of this difference will be the side *B C*.

3rd. The lines *A C*, *B C* being given, to find *A B*.

Rule.—*Square the lines A C and B C, and take their difference; the square root of this difference will be the side A B.*

FORMULÆ.

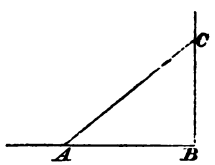
1st. $AC = \sqrt{AB^2 + BC^2}$.

2nd. $BC = \sqrt{AC^2 - AB^2}$.

3rd. $AB = \sqrt{AC^2 - BC^2}$.

Example 15.—Section 6.

Let AC be the position of the ladder. When C slides to B, then A has slid 15 feet; hence $AB = 54 - 15 = 39$ feet.



Therefore $BC = \sqrt{AC^2 - AB^2} = \sqrt{54^2 - 39^2} = 37$ ft. 4 in.

EXAMPLES.

Section 1.

1. Given $BC = 50$ feet, $AB = 80$ feet; find AC.
2. Given $AB = 94$ feet, $AC = 194$ feet; find BC.
3. Given $AC = 144\frac{1}{2}$ feet, $BC = 124$ feet; find AB.

Section 2.

4. Given $AC = 84$ yards, $AB = 48$ yards; find BC.
5. Given $BC = 534\frac{1}{2}$ feet, $AB = 435\frac{1}{2}$ feet; find AC.
6. Given $AB = 194$ feet, $AC = 491$ feet; find BC.

Section 3.

7. Given $BC = 34\frac{1}{4}$ feet, $AC = 43\frac{1}{2}$ feet; find AB
8. Given $AC = 56\frac{1}{8}$ feet, $AB = 16\frac{1}{8}$ feet; find BC.
9. Given $AB = 58\frac{5}{8}$ feet, $BC = 85\frac{7}{12}$ feet; find AC.

Section 4.

10. Given $BC = 535\frac{5}{12}$ feet, $AC = 553\frac{5}{12}$ feet; find AB.
11. Given $BC = 4$ feet, $AB = 3$ feet; find A C.
12. *Given* $BC = 2\,AB$; required A C, when A B = 10.

Section 5.

13. The sum of $A\,B$ and $B\,C$ is 36 feet, and their difference is 14 feet; required $A\,C$.

14. A sheer leg, whose length is 104 feet, is inclined to the horizon, so that the distance from its base to the foot of a perpendicular from its top is 26 feet; required its height.

Section 6.

15. A ladder, 54 feet long, is placed with one end against an upright wall, and the other at a certain distance from the foot of the wall; the ladder is then observed to slide 15 feet on the ground when its top reaches the ground; required the height of the wall.

16. The side of a square is $84\frac{1}{8}$ feet; required the length of its diagonal.

Section 7.

17. The length of a beam of a ship is 24 feet 6 inches, and the distance of the perpendicular from the middle of the beam to the apron is 14 feet 8 inches; required the length of the horning lines.

18. The perpendicular on the stem of a vessel is 30 feet 3 inches, and a distance of 25 feet 6 inches is set up on the perpendicular; find the hypothenuse.

The following require the application of Algebra.

Section 8.

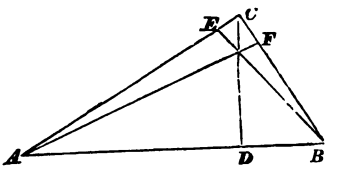
19. A ladder, 15 feet long, is placed with one end against an upright wall, and the other on a horizontal line; while the upper end falls to a height of 3 feet the lower one slides 3 feet; find the height of the ladder in its first position.

20. A ladder, 45 feet long, is placed with one end against an upright wall, and the other on a horizontal line; the foot of the ladder is observed to slide so that its distance from the wall is now doubled, and the height of the top from the foot of the wall is now the same distance as the foot of the ladder was from the wall in its first position; how far has the top slid?

21. A ladder, 20 feet long, is placed with one end against an upright wall, and the other on a horizontal line; required the position of the ladder so that when the foot of it slides *2 feet on the* ground the top may slide 1 foot down the wall.

Problem 2.

Given, the three sides of a triangle; to find the perpendicular from any angle to its opposite side.



Rule.—Add the sides $A\ C$ and $B\ C$ together, and call this the first *sum*.

Subtract the line $B\ C$ from $A\ C$, and call this the *difference*.

Multiply the *sum* and *difference* together, and add the product to the *square* of the side $A\ B$.

Divide this last *sum* by twice the side $A\ B$, and the quotient will be the segment $A\ D$.

Then, the triangle $A\ C\ D$ being a right angled triangle, the perpendicular $C\ D$ can be found by Problem 1.

The three perpendiculars $A\ F$, $B\ E$, $C\ D$ will always pass through the same point.

Formula.

Put $A\ B = c$, $A\ C = b$, $B\ C = a$.

$$\therefore A\ D = \frac{1}{2\ c}(b^2 + c^2 - a^2) \text{ and } B\ D = \frac{1}{2\ c}(a^2 + c^2 - b^2).$$

$$\text{And, } C\ D = \sqrt{b^2 - A\ D^2} = \sqrt{a^2 - B\ D^2}\ .$$

The segments $A\ D$, $B\ D$, may also be found by the following rule,—

$$A\ B : A\ C + B\ C :: A\ C - B\ C : A\ D - B\ D.$$

One half of this difference being added to, and subtracted from half the line $A\ B$, will give the segments $A\ D$ and $B\ D$.

Example 5.—Section 4.

$$A\ D = \frac{b^2 + c^2 - a^2}{2\ c} = \frac{49 + 64 - 36}{16} = \frac{77}{16},$$

$$\text{And } B\ D = 8 - \frac{77}{16} = \frac{51}{16}.$$

$$\text{Then, } C\ D = \sqrt{A\ C^2 - A\ D^2} = \sqrt{49 - \frac{77^2}{16^2}} = 5.083.$$

Examples.

Section 1.

1. The base $A\,B = 16$ feet, $A\,C = 11$ feet, and $B\,C = 8$ feet; required the segments and perpendicular on $A\,B$.
2. The two sides of a triangle are 2 and 3, and the base 4; reguired the segments and perpendicular with respect to each angle.

Section 2.

3. The three sides of a triangle are 14, 9, and 12 feet; required the segments and perpendicular with respect to each angle.

Section 3.

4. The three sides of a triangle are 24, 18, and 15 feet; required the segments and perpendicular with respect to each angle.

Section 4.

5. The three sides of a triangle are 8, 7, and 6 feet; required the segments and perpendicular with respect to each angle.

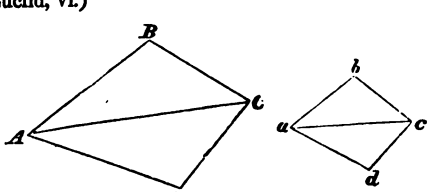
Problem 3.

In two similar figures there is given two corresponding dimensions, one in each figure, and also another dimension in one figure; to find its corresponding dimension in the other.

N.B.—"Similar figures are those which have their several angles equal, each to each, and the sides about the equal angles proportional." (Euclid, vi.)

Rule.—

$A\,D : a\,d :: D\,C : d\,c.$



The same proportion holds in respect to any other corresponding lines in the similar figures.

Example 18.—Section 6.

28 miles per hour $= 28 \times 5280 = 147840$ feet per hour.

Then, 1071 : 7 :: 147840 : height required $= 966{\cdot}27$ feet.

Examples.

Section 1.

1. The line $A\,B = 9$ feet, $B\,C = 15$ feet, and $a\,b = 3$ feet; required $b\,c$.
2. The line $A\,B = 30$ feet, $B\,C = 20$ feet, $C\,D = 10$ feet, $A\,D = 8$ feet, and $a\,b = 18$ feet; find the lengths of $b\,c$, $c\,d$, $a\,d$.

Section 2.

3. The length of the shadow of St. Paul's is 52 yards, when the shadow of the Monument is 24 yards; what will be the length of the shadow of the Monument when the shadow of St. Paul's is 62 yards?
4. The length and breadth of a town are $3\frac{1}{2}$, $2\frac{3}{4}$ miles respectively; the length on a plan is 4 feet $5\frac{1}{2}$ inches; what is the breadth?
5. The circumference of a circle, whose diameter is a unit, is 3·1416; find the circumference of another circle whose diameter is 49.

Section 3.

6. What is the circumference of a rope whose diameter is $3\frac{1}{2}$ inches?
7. Find the circumference of an iron bolt whose diameter is $1\frac{1}{2}$ inch.
8. The circumference of an iron bolt is 6 feet $9\frac{5}{8}$ inches; what it its diameter?
9. The circumference of an iron axle is 4 feet $8\frac{1}{4}$ inches; find its diameter.

Section 4.

10. The length of a shadow of any object is 84 feet, while the shadow of a 3 feet cane is 2 feet 6 inches; what is the height of the object?
11. The circumference of a barrel is 24 feet $1\frac{1}{3}$ inch; find its diameter.
12. The circumference of a circle is 116 feet $2\frac{7}{8}$ inches; what is its diameter?
13. The circumference of a circle is 154 feet $11\frac{7}{8}$ inches; find its diameter.

Section 5.

14. The two wheels of a coach are 4 feet 6 inches, and 4 feet in *diameter*; how many times will each turn in a journey *20 miles?*

Section 5—*(continued)*.

15. The circumference of a steam-engine cylinder is 13 feet 1 inch ; find its diameter.

16. The circumference of a well is 16 feet 9 inches ; what is its diameter.

Section 6.

17. A railway-train moves, up an inclined plane of 3 in 396, at the rate of 37 miles an hour ; what vertical height will it ascend per minute ?

18. A railway-train moves, up an inclined plane of 7 in 1071, at 28 miles per hour ; what vertical height will it ascend per hour ?

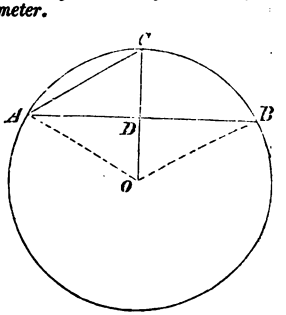
19. An inclined plane rises 18 yards in 3½ miles ; in how many yards will it rise 1 yard ?

20. Two vessels, 7½ miles distant, the one sails in a direction of 1 to 34, and the other 1 to 43, with the line joining their original positions ; how far will each sail before it crosses the other's direction.

PROBLEM 4.

Given the chord A B and versed sine C D of an arc of a circle; to find its diameter.

Rule.—Divide the square of the chord *A B* by four times the versed sine *C D*, to the quotient add the versed sine *C D*, and the sum will be the diameter.



FORMULA.

Put the chord = *c*, and the versed sine = *v*.

$$\therefore \text{the diameter} = \frac{c^2}{4v} + v = \frac{A\,C^2}{v}$$

Example 7.—Section 3.

$$\textit{Radius} = \frac{c^2}{8v} + \frac{v}{2} = \frac{208^2}{8 \times 16{\cdot}75} + \frac{16{\cdot}75}{2} = 331{\cdot}2 \text{ feet.}$$

Examples.

Section 1.

1. The chord of an arc is 12 feet, and the versed sine is 2 feet; find the diameter.
2. The chord of an arc is 18 feet 6 inches, and the versed sine 5 feet 6 inches; find the diameter.
3. The span of a bridge is 30 feet 6 inches, and its height in the middle, from the springing line, is 8 feet 9 inches; find the radius of the centering.

Section 2.

4. The span of the middle arch of Dunkeld-bridge is 90 feet, and rise of arch 30 feet; required the radius of the arch.
5. The bridge at Wearmouth is 240 feet span, and rises 30 feet in the middle; what is the radius of the arch?
6. The span of the Buildwas-bridge, near Colebrook Dale, is 130 feet, and height of the versed sine 17 feet; required the radius of the arch.

Section 3.

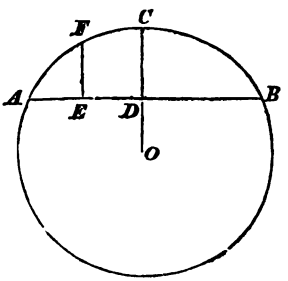
7. The span of the wooden bridge on the Regnitz, in Germany, is 208 feet, with a rise of 16·9 feet; required the radius of the arch.
8. The span of the bridge over the Don, 7 miles above Aberdeen, is 109¼ feet, the rise 13½ feet; required the radius.
9. The bridge at Walton on the Thames has a span of 130 feet, and a rise of 27 feet; required the radius.

Section 4.

10. The bridge at Ettringen, over the Wertach, has a span of 138 feet, and a radius of curvature of 305 feet; find the rise in the middle.
11. The bridge of Augsburg, on the Lech, has a span of 114 feet, and a radius of curvature of 158 feet; find the rise in the middle.
12. The bridge of Freysingen, on the Isar, has a span of 153 feet, and the radius of curvature 246 feet; find the rise of the arch in the middle.

Problem 5.

Given the chord A B, the versed sine C D of a circle, the distance D E; to find the ordinate E F.



Rule. — Find the radius by Problem 4.

(1.) — Subtract the versed sine from the radius.

(2.)—To the radius add and subtract the distance $E D$; multiply this sum and difference together, and extract the square root of the product.

Subtract (1) from (2), and this difference will be the ordinate $E F$.

Formula.

Put r = radius, the other quantities as in Problem 4.

$$\therefore E F = \sqrt{(r - D E)(r + D E)} - (r - v).$$

Examples.

Section 1.

1. The chord and versed sine of an arc of a circle are 14 feet and 6 feet respectively; find the ordinates at 1, 2, and 3 feet from the middle of the chord.

Section 2.

2. A railway bridge, single line, is 36 feet span and 9 feet rise: the diameter of the funnel of a locomotive is 1 foot 6 inches, and the level of the rail below the springing of the arch is 16 feet; what is the greatest height of the engine and funnel the bridge will allow?

Section 3.

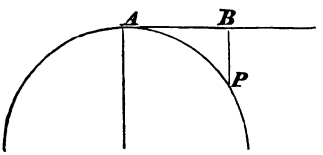
3. A tunnel for a double line of railway is 22 feet span, and rise 18 feet; the rails are 5 feet below the springing line, and the outer one is 8 feet 1½ inch from the centre of the tunnel; find the highest carriage which the tunnel will take.

Problem 6.

From a given distance A B along the tangent to a circle, parabola, and ellipse; find the deflection B P to the curve.

Circle.

Rule.—Take the sum and difference of the radius and given distance (*A B*), multiply them together and extract the square root; then subtract this quantity from the radius, and the result will be the deflection.



Parabola.

Rule.—Square the distance *A B*, and divide this result by four times the distance from the vertex to the focus, and the quotient will be the deflection.

Square half the chord of the parabola, and divide the result by the height or versed sine of the curve; the quotient is four times the focal distance.

Ellipse.

Rule.—Take the sum and difference of the semiaxis minor and the distance (*A B*), multiply them together and extract the square root of the product, subtract this quantity from the semiaxis minor; multiply the result by the semiaxis major, and divide the product by the semiaxis minor, and the quotient will be the deflection.

Formulæ.

Let $A\ B = t$; $B\ P = d$; r = radius of circle; f = focal distance in parabola; a = semiaxis major; b = semiaxis minor of the ellipse.

For the circle, $d = r - \sqrt{r^2 - t^2}$.

For the parabola, $d = \frac{t^2}{4f}$.

For the ellipse $d = a - \frac{a}{b}\sqrt{b^2 - t^2}$.

When the deflection of curves is small, they may be considered portions of a parabola without producing any appreciable error.

Example 8.—Section 6.

$\frac{19^2}{\cdot 5} = 722$ feet, which is four times the focal distance.

Then, $\frac{49}{722} = .0678$ feet, the offset at 7 feet.

EXAMPLES.

Section 1.

1. Find the offsets, at the end of every two chains, to 10 chains from the tangent to a railway curve whose radius is 184 chains.

Section 2.

2. The distance from the vertex to the focus of a parabola is 7 feet 6 inches; find the deflection from a distance of 18 feet 9 inches along the tangent.
3. The beam of a first rate is 42 feet, and cambered 9 inches in the centre; what are the offsets at the distance along the tangent of 5, 10, and 15 feet, considering the curve as a parabola?

Section 3.

4. From the extremity of the major axis of an ellipse whose diameters are 18 and 14 feet, a distance of 5 feet 6 inches is measured; what is the deflection to the curve at this point?
5. The radius of the earth is 3979 miles; find the offset at the distance of 5 miles, supposing the earth to be a sphere.

Section 4.

6. The equatorial diameter of the earth is 7924.873 miles, and the polar diameter is 7898.634 miles; find the offsets from a tangent to the meridian at the equator, at the distances of 4, 16, and 20 miles.

Section 5.

7. The height of the hull and mast of a vessel is 100 feet; to what distance must it sail before it disappears below the horizon? Find the same when the vessel is 200 feet.

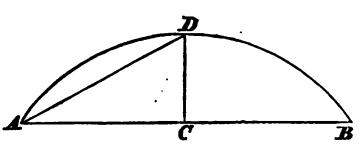
Section 6.

8. The beam of a first-rate is 38 feet, and cambered 6 inches in the middle; what are the offsets at the distances along the tangent of 7, 12, and 14 feet, considering the curve a parabola from the middle?

PROBLEM 7.

Given the base and height of the segment of a circle, to find the length of its arc $\overset{\frown}{ADB}$.

1st.—When the radius and degrees in the arc are given.



Then, 180 : degrees in $\overset{\frown}{ADB}$ :: 3.1416 × radius : to the length of the arc $\overset{\frown}{ADB}$.

2nd.—When the base (AB), and height or versed sine (CD) are given.

Find the radius by Problem 4.

Divide (CD) the height by (AC) the base, and opposite to this quotient in Table (A) there will be found one-fourth of the degrees in the arc $\overset{\frown}{ADB}$.

Find the length corresponding to these degrees by case the first.

TABLE (A).

1st Col.	2nd Column.	3rd Col.	1st Col.	2nd Column.	3rd Col.
Degrees.	Tangent.	$\frac{1}{3438}$ of the difference.	Degrees.	Tangent.	$\frac{1}{3438}$ of the difference.
..30	·0087269	2909	23..	·4244748	3445
1..	·0174551	2910	23..30	·4348124	3472
1..30	·0261859	2911	24..	·4452287	3499
2..	·0349208	2913	24..30	·4557263	3527
2..30	·0436609	2915	25..	·4663077	3556
3..	·0524078	2918	25..30	·4769755	3586
3..30	·0611626	2921	26..	·4877326	3616
4..	·0699268	2925	26..30	·4985816	3647
4..30	·0787017	2929	27..	.5095254	3680
5..	·0874887	2933	27..30	·5205671	3714
5..30	·0962890	2938	28..	·5317094	3748
6..	·1051042	2943	28..30	·5429557	3784
6..30	·1139356	2949	29..	·5543091	3821
7..	·1227846	2955	29..30	·5657728	3859
7..30	·1316525	2962	30..	·5773503	3898
8..	·1405408	2970	30..30	·5890450	3938
8..30	·1494510	2977	31..	·6008606	3980
9..	·1583844	2986	31..30	·6128008	4023
9..30	·1673426	2995	32..	·6248694	4067
10..	·1763270	3004	32..30	·6370703	4112
10..30	·1853390	3014	33..	·6494076	4159
11..	·1943803	3024	33..30	·6618856	4207
11..30	·2034523	3035	34..	·6745085	4257
12..	·2125566	3046	34..30	·6872810	4308
12..30	·2216947	3058	35..	·7002075	4361
13..	·2308682	3070	35..30	·7132931	4416
13..30	·2400788	3083	36..	·7265425	4473
14..	·2493280	3096	36..30	·7399611	4531
14..30	·2586176	3110	37..	·7535541	4591
15..	·2679492	3125	37..30	·7673270	4653
15..30	·2773245	3140	38..	·7812856	4716
16..	·2867454	3156	38..30	·7954359	4782
16..30	·2962135	3172	39..	·8097840	4850
17..	·3057307	3189	39..30	·8243364	4921
17..30	·3152988	3207	40..	·8390996	4994
18..	·3249197	3225	40..30	·8540807	5069
18..30	·3345953	3244	41..	·8692867	5146
19..	·3443276	3263	41..30	·8847253	5226
19..30	·3541186	3283	42..	·9004040	5309
20..	·3639702	3305	42..30	·9163312	5394
20..30	·3738847	3326	43..	·9325151	5483
21..	·3838640	3348	43..30	·9489646	5574
21..30	·3939105	3371	44..	·9656888	5669
22..	·4040262	3395	44..30	·9826973	5767
22..30	·4142136	3420	45..	1·0000000	5869

Approximate Rule (usually called Huygens' Theorem).

From eight times the chord of half the arc, subtract the chord of the whole arc, and one third of the remainder will be the length of the arc, nearly.

FORMULA.

$$\widehat{ADB} = \frac{8\,AD - AB}{3}$$

EXAMPLE.

1. Find the length of the arc, when the base is 30 feet, and height 3 feet.

$$\text{Radius} = \frac{15^2 + 9}{6} = \frac{225 + 9}{6} = \frac{234}{6} = 39 \text{ feet. By Problem 4.}$$

$$\text{Again, } \frac{3}{15} = \frac{1}{5} = \frac{.2000000}{.1943803}\Big\{\text{the number next to .2 in Table (A.) and opposite to 11 degrees,}$$

$$56197$$

To find the minutes and seconds.

3024)	56197	(18.584	The division 3024 is opposite 11° in Table (A).
	3024	60	
	25957	35.040	
	24192		
	17650		
	15120		
	25300		
	24192		
	12080		

Therefore the degrees of the arc of the segment are

$$4\ (11^\circ\ 18'\ 35'') = 45^\circ\ 14'\ 20'' = 45.239^\circ.$$

$\therefore$ 180 : 45.239 :: 3.1416 × 39 : 30.793 feet, the length of the arc.

The approximate rule by Huygens' Theorem, gives 30.879 feet.

Section 1.

1. *Find the length* of the arc, when the base and height are *40 and 4 feet.*

Section 1—*(continued)*.

2. Find the length of the arc, when the base and height are 35 and 5 feet.

Section 2.

3. The bridge at Wearmouth has a rise of 30 feet, and a span of 240 feet; find the length of the intrados.

4. Vauxhall-bridge has a rise of 11 feet 6 inches, and a span of of 78 feet; find the length of the intrados.

5. Southwark-bridge has a rise of 24 feet, and a span of 240 feet; find the length of the intrados.

Section 3.

6. The Dunkeld-bridge crosses the Tay with a span of 90 feet, and a rise of 30 feet; find the length of the intrados.

7. The bridge of Augsburg, on the Lech, has a span of 114 feet with 10 feet 6 inches rise; find the length of the intrados.

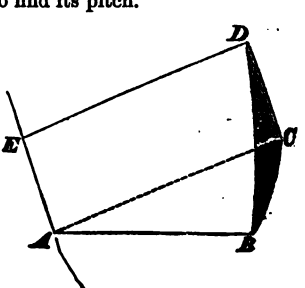
Section 4.

8. The bridge of Freysingen, on the Isar, has a span of 153 feet, with a rise of 11 feet 6 inches; find the length of the intrados.

9. The bridge of Sharding, on the Rolt, has a span of 194 feet, with a rise of 18 feet 8 inches; find the length of the intrados.

PROBLEM 8.

Given the radius, length of helix at the extremity of the screw blade, the length of the screw; to find its pitch.



The line $A\ E$ is the axis of the screw, $A\ B\ D\ E$ is the screw blade, $B\ D$ is the helix, $A\ B$ is the radius, and $D\ C = A\ E$ is the length of the screw.

RULE.

(1). Multiply the radius by the length, and by twice 3.1416.

(2.) Take the square root of the product of the sum and difference of the helix ($B\ D$), and the length ($A\ E$).

(3.) Divide (1) by (2), and the quotient will be the pitch.

FORMULA.

Put r = radius, l = length, H = length of helix $B\ D$, $3.1416 = \pi$.

Then, pitch $= \dfrac{2\pi r l}{\sqrt{H^2 - l^2}}$.

EXAMPLES.

Section 1.

1. The radius, length, and length of helix of a screw propeller are 7 feet, 3 feet, and 5 feet; find the pitch.

2. The radius, length, and length of helix of a screw propeller are 9 feet, 3 feet 6 inches, and 6 feet; find the pitch.

Section 2.

3. The radius, length, and length of helix of a screw propeller are 8 feet, 3 feet 4 inches, and 8 feet; find the pitch.

4. The radius, length, and length of helix of a screw propeller are 5 feet, 1 foot 3 inches, and 4 feet 6 inches; find the pitch.

PROBLEM 9.

Given the speed of a vessel and the number of revolutions made by the propeller in a minute, to find the slip of the screw per cent.

When the vessel advances the length of the pitch of the screw during one revolution, the vessel is moving as fast as the screw.

When the vessel advances a distance greater than the pitch of the screw, the vessel is moving faster than the screw.

When the vessel advances a distance less than the pitch of the screw, the vessel is moving slower than the screw.

The distance, which is the difference between the pitch and the space through which the vessel is propelled during one revolution of the screw, is called the slip of the screw.

The slip of a screw, like the speed of a moving body, may be measured in various ways, as feet per hour, miles per hour, knots per hour, &c.

A knot = 6080 feet. A mile = 5280 feet.

Example.

Her Majesty's ship *Ajax* was propelled at the rate of 7.147 knots per hour by a screw whose pitch was 17 feet 11 inches, and making 48 revolutions per minute. Find the slip of the screw per cent. in knots per hour.

The speed of the screw in knots per hour

$$= \frac{60 \times 48 \times 17.91666}{6080} = 8.486.$$

Then, $8.486 - 7.147 = 1.339$ the slip in knots per hour.

Hence, $8.486 : 1.339 :: 100 : 15.78$ slip per cent.

This example will indicate the process of finding the slip per cent.

Rule 1.

1st. Multiply three times the pitch of the screw by the number of revolutions per minute, divide this product by 304, and the quotient will be the speed of the screw in knots per hour.

2nd. The difference between the speed of the screw and the speed of the vessel, in knots per hour, is the slip of the screw in knots per hour.

3rd. Then, the speed of the screw : to slip of screw :: 100 : to slip per cent.

Formula.

Let K = the speed of vessel in knots per hour.

n = number of revolutions of screw per minute.

p = pitch of screw in feet.

Then, $\frac{60\,n\,p}{6080} = \frac{3\,n\,p}{304} =$ } the speed of the screw in knots per hour.

Then, $\frac{3\,n\,p}{304} - K =$ slip of screw in knots per hour.

$$\frac{3\,n\,p}{304} : \left\{\frac{3\,n\,p}{304} - K\right\} :: 100 : 100 \left\{1 - \frac{304\,K}{3\,n\,p}\right\} \text{ the slip per cent.}$$

Therefore, $100 \left\{1 - \frac{304\,K}{3\,n\,p}\right\} =$ slip per cent.

Rule 2.

Divide 304 times the speed of the vessel by 3 times the product of the pitch, and the number of revolutions of the screw per minute; subtract this quotient from 1, and 100 times the difference will be the slip per cent.

Examples.

Section 1.

1. Her Majesty's ship *Arrogant* was propelled 8.295 knots per hour by a screw of 15 feet pitch, and 55.5 revolutions per minute; find the slip per cent.
2. The *Blenheim* was propelled at 5.816 knots per hour, with a screw of 20 feet pitch, and 43 revolutions per minute; find the slip per cent.
3. The *Dauntless* was propelled at 7.366 knots per hour, with a screw of 18 feet pitch, and 55.3 revolutions per minute; find the slip per cent.
4. The *Desperate* was propelled at 9.432 knots per hour, with a screw of 14 feet pitch, and 74.69 revolutions per minute; find the slip per cent.

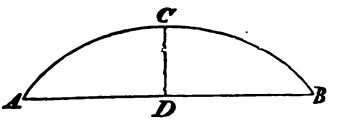
Find the slip per cent. in the following:—

Question.	Speed of vessel in knots per hour.	Pitch of screw in feet.	Number of revolutions per minute.	Name of vessel.
		Section 2.		
5.	10.254	15	78	*Encounter.*
6.	10.537	8	182.8	*Dwarf.*
7.	13.324	8	258	*Fairy.*
8.	7.5	20	50	*Hogue.*
		Section 3.		
9.	9.137	5	254.12	*Minx.*
10.	9.494	17	68.8	*Niger.*
11.	7.106	$12\frac{8}{9}$	68	*Phœnix.*
12.	6.369	7	99.15	*Plumper.*
		Section 4.		
13.	10.074	11	104	*Rattler.*
14.	8.096	9	119.25	*Rifleman.*
15.	9.782	9	140.7	*Sharpshooter.*
16.	9.166	18	66	*Termagant.*
17.	9.605	$16\frac{1}{2}$	66.5	*Vulcan.*

Miscellaneous Formulæ.

To find the circumference of an Ellipse.

Let a and b be the conjugate and transverse diameters, and $d = 1 - \frac{b^2}{a^2}$.

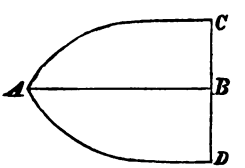


$$\text{Circumference} = \pi a \left\{ 1 - \frac{d}{2^2} - \frac{3\,d^2}{2^2\,4^2} - \frac{3^2\,5\,d^3}{2^2\,4^2\,6^2} - \frac{3^2\,5^2\,7d^4}{2^2\,4^2\,6^2\,8^2} - \&c. \right\}$$

$$\text{Approximate rule} = \frac{\pi}{2}(a + b).$$

To find the circumference of a Parabola.

Put m = focal distance, $A\,B = a$.



$$\text{Arc } C\,A\,D = 2\sqrt{a^2 + m\,a} + m \log. \left(\frac{2\,a + m + 2\sqrt{a^2 + m\,a}}{m} \right)$$

$$= 2\sqrt{C\,B^2 + \frac{4\,a^2}{3}} \quad \text{approximately.}$$

To find the circumference of a Hyperbola.

Let a and b be the transverse and conjugate axis, and x the abscissa, and y the ordinate, corresponding to the arc.

$$\textit{Then, the arc} = \frac{15\,a\,b^2 + (19\,a^2 + 21\,b^2)\,x}{15\,a\,b^2 + (9\,a^2 + 21\,b^2)\,x} \times y \text{ nearly.}$$

The Cycloid.

The arc of the cycloid is four times the diameter of its generating circle. Discovered by Wren.

Evolute of an Ellipse.

The fourth part of the length of an evolute of an ellipse

$$= \frac{a^3 - b^3}{a\,b}$$

where a and b are the semi axis.

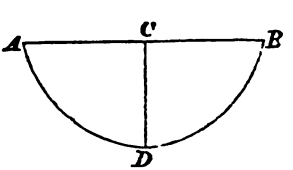
Involute of a Circle.

The arc of the involute is equal to the square of the string unwound from the circle, divided by the diameter of the circle.

The Catenary.

The arc $A\,D\,B = 2\sqrt{D\,C^2 + 2\,D\,C.t}$ where t is the tension at D.

$$B\,C = A\,C = t \log. \left\{ \frac{D\,C + t + \sqrt{D\,C^2 + 2\,D\,C.t}}{t} \right\}$$



MENSURATION OF SUPERFICES OR SURFACES.

TABLE OF SQUARE UNITS.

144 square inches... = one square foot.
9 square feet..... = one square yard.
30¼ square yards ... = one square pole, rod, or perch.
40 poles ..... = one rood.
4 roods ..... = one acre.
640 acres ..... = one square mile.

One acre = 10 square chains = 4840 square yards = 43560 square feet.

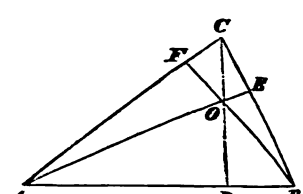
Problem 1.

Find the area of a triangle.

Rule 1.

When any side, and the perpendicular from the opposite angle on that side are given.

Multiply the perpendicular by half the base, and the product will be the area.



Rule 2.

When three sides are given. Add the three sides together and divide the sum by 2 ; *call this half the sum.*

From this half sum subtract each side separately ; call these the three remainders.

Multiply the half sum and the three remainders together, then *extract the* square root of the product, and it will be the area of *the triangle.*

Formulæ.

Let $A\,B = c$, $A\,C = b$, $B\,C = a$.

Draw the three perpendiculars $C\,D$, $A\,E$, $B\,F$.

These three perpendiculars, when correctly drawn, will pass through the same point O.

Put A = area of triangle, and s = half the sum of the three sides.

$$\therefore A = \frac{c \times C\,D}{2} = \frac{a \times A\,E}{2} = \frac{b \times A\,C}{2}$$

$$\text{Put, } s = \frac{a + b + c}{2}$$

$$\text{Then, } A = \sqrt{s\,(s-a)\,(s-b)\,(s-c)}$$

Example 44.—Section 18.

$$s = \frac{a+b+c}{2} = \frac{\begin{matrix}\text{yds.}\ \text{ft.} & \text{yds.}\ \text{ft.} & \text{yds.}\\ 24\ \ 2 & +\ 28\ \ 1 & +\ 34\end{matrix}}{2} = 43.5 \text{ yds.}$$

$$s - a = \frac{113}{6} \text{ yds.}; \quad s - b = \frac{91}{6} \text{ yds.}; \quad s - c = \frac{19}{2} \text{ yds.}$$

$$\text{Area} = \sqrt{\frac{87}{2} \times \frac{113}{6} \times \frac{91}{6} \times \frac{19}{2}} = \frac{1}{12}\sqrt{16997799}$$

$$= 343.5698 \text{ square yards.}$$

Then, 343.5698×26 shillings $= 446l.\ 12s.\ 9\frac{3}{4}d.$

Examples.

Section 1.

1. The base of a triangle is 84 yards, and the perpendicular 15 yards; required its area in square yards and square feet.

2. The base of a triangle is 8 feet 6 inches, and the perpendicular is 3 feet 10 inches; how many square yards does it contain?

3. The base of a triangular field is 8 chains 54 links, and the perpendicular is 3 chains 87 links; how many acres does it contain?

4. *The base of a triangle is 132 feet 8½ inches, and its perp.*
84 feet 7½ inches; how many square yards does it contain

Section 2.

5. A triangular field, whose base is 4.76 chains, has a perpendicular of 2.34 chains; how many acres does it contain?

6. The base of a triangle is 57 chains and 34 links, and its perpendicular is 4 chains 67 links; find its area.

7. The base of a triangle is 52.21 chains, and its perpendicular 61.71 chains; required its area.

Section 3.

8. A triangular plot of ground measures along its base 42.24 chains, its perpendicular being 32.74 chains; what area does it contain?

9. A field, forming a triangle, whose base and perpendicular measures 67.41 chains and 11.11 chains; what annual rent does it produce at 2*l.* 10*s.* per acre?

10. A triangular field, whose base and perpendicular are 42.63 chains and 24.42 chains, is sold at 96*l.* 3*s.* 6*d.* per acre; what is the amount of the purchase money?

Section 4.

11. The diagonal of a rectangular field is 18.56 chains, and one of the sides 10 chains 36 links; what is the price of the field at 104*l.* 16*s.* 8*d.* per acre?

12. What is the area of a triangular garden whose base is 104 yards, and perpendicular 78 yards 2 feet?

13. The base of an equilateral triangle is 34 feet 6 inches; find its area in square yards.

Section 5.

14. The equal sides of an isosceles triangle are each 25 feet $7\frac{1}{2}$ inches, and the base 22 feet 6 inches; find its area in square yards.

15. The hypothenuse of a right angled triangle is 6 feet $7\frac{1}{2}$ inches, and its base 3 feet $4\frac{3}{4}$ inches; find its area.

Section 6.

16. The area of a triangle is equal to the sum of the base and perpendicular, and the base is 3 feet; find the perpendicular and area.

17. The area of a triangle is equal to the difference between the *base and* perpendicular, and the perpendicular is 14 feet; *find the base* and area.

Section 7.

18. The base of a triangle is equal to the square of its area, and the perpendicular is 12 feet; find the base.

19. The area of a triangle is equal to the base divided by its perpendicular, and the base is 8 feet; what is the area?

Section 8.

20. The area of a triangle is equal to the perpendicular added to 7, and the base is equal to 5; find the perpendicular and area.

21. The area of a triangle is equal to the base added to 14, and the perpendicular is equal to the base added to 29; find the area.

22. The area of a triangle is 6 square feet, and the base is equal to the sum of the area and perpendicular; find the perpendicular.

Section 9.

23. Find the area of a triangle, whose three sides are 8, 16, and 14 feet.

24. Find the area of a triangle, whose three sides are 18, 20, and 25 feet.

25. Find the area of a triangle, whose three sides are 120, 150, and 180 feet.

Section 10.

26. Find the area of a triangle, whose three sides are 14, 9, and 12 feet.

27. Find the area of a triangle, whose three sides are 24, 18, and 15 feet.

28. Find the area of a triangle, whose three sides are 8, 7, and 6 feet.

Section 11.

29. Find the area of a triangle, whose three sides are 15, 12, and 8 feet.

30. Find the area of a triangle, whose three sides are 14, 10, and 6 feet.

31. Find the area of a triangle, whose three sides are $4\frac{1}{2}$, $5\frac{1}{2}$, and $6\frac{1}{2}$ feet.

Section 12.

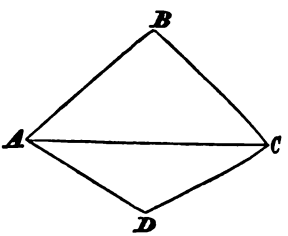
32. Find the area of a triangle, whose three sides are $8\frac{1}{2}$, $15\frac{1}{2}$, and $13\frac{1}{4}$ feet.

33. Find the area of a triangle, whose three sides are 15, $18\frac{1}{2}$, and $13\frac{1}{2}$ feet.

Section 13.

34. Find the area of a garden of the form

$A\,B = 19$ yards
$B\,C = 25$ „
$A\,C = 30$ „
$A\,D = 15$ „
$D\,C = 24$ „



Section 14.

35. Find the acres in a field of the above form, $A\,B = 15$, $B\,C = 24$, $A\,C = 30$, $A\,D = 45$, and $D\,C = 35$ chains.

Section 15.

36. The base of a triangle is 184 feet, and the perpendicular upon it from the opposite angle is 18 feet 6 inches; find the square yards it contains.

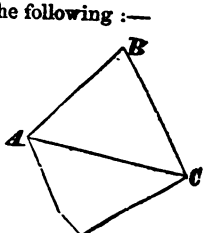
37. What is the area of an equilateral triangle, whose three sides are 186 feet?

38. The area of a triangle is 134 square feet, and its base is 18 feet $7\frac{1}{2}$ inches; find the perpendicular.

Section 16.

39. Find the area of a field, whose form is the following :—

$A\,B = 56$ yards
$B\,C = 64$ „
$A\,C = 94$ „
$D\,C = 52$ „
$A\,D = 46$ „



Section 17.

40. Find the area of a triangle, in acres, whose sides are $98\frac{1}{2}$, $84\frac{1}{2}$, and $86\frac{1}{2}$ yards.

41. Find the area of a triangle, in acres, whose sides are 108, 144, and 216 yards.

42. Find the area of a triangle, in acres, whose sides are $20\frac{1}{2}$, $30\frac{1}{2}$, and $40\frac{3}{4}$ feet.

Section 18.

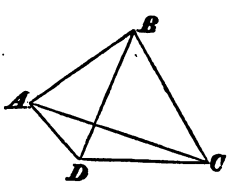
43. The sides of an equilateral triangular field are each $8\frac{1}{2}$ chains; what is its value at 84*l.* per acre?

44. A triangular plot of ground measures along its three sides, 24 yards 2 feet, 28 yards 1 foot, and 34 yards; what is its value at 26*s.* per square yard?

PROBLEM 2.

To find the area of a trapezium, and also of any rectilineal figure.

Divide it into two triangles by either of the diagonals *A C* or *B D*, then find the area of each of the triangles by Prob. 1.

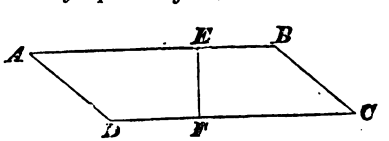


Any rectilineal figure of any number of sides can be divided into triangles whose areas may be found by Prob. 1.

PROBLEM 3.

To find the area of a parallelogram.

Rule.—Multiply the base (*A B*) by the perpendicular (*E F*) between the sides, and the product will be the area.



FORMULA.

$$\text{Area} = A\,B \times E\,F.$$

Example 14.—Section 7.

$$\frac{\text{yds. } 132 + \text{yds. } 78}{2} = \frac{210}{2} \text{ yds.} = 105 \text{ yds.} = \frac{105}{22} \text{ chains, } = 4{\cdot}7727 \text{ chains.}$$

Then, $13{\cdot}14 \times 4{\cdot}7727 = 62{\cdot}713$ square chains, $= 6{\cdot}2713$ acres = 6 acres, 1 rood, 3 perches, $12\frac{1}{4}$ square yards.

EXAMPLES.

Section 1.

1. The two sides of a rectangle are 14 feet $6\frac{1}{2}$ inches, and 3 feet $5\frac{1}{2}$ inches; how many square feet does it contain?
2. A garden whose form is a parallelogram, one side of which is $3\frac{1}{2}$ chains, and the perpendicular between the sides $2\frac{1}{4}$ chains; how many roods does it contain?
3. A balk of timber is 36 feet 8 inches long, and 2 feet 6 inches wide; what is the value of it at $8\frac{1}{2}d.$ per square foot?

Section 2.

4. An oblong room is 36 feet $8\frac{1}{2}$ inches long, and 18 feet $7\frac{1}{4}$ inches wide; how many yards of carpet, 2 feet $6\frac{1}{2}$ inches wide, will cover it? and what is the value of the carpet at 5*s.* 10*d.* per yard?
5. A square field containing 5 acres is to be fenced round at the rate of 2*s.* 6*d.* per lineal yard; what is the price of fencing it?

Section 3.

6. An oblong field is 8 chains 27 links long, and 5 chains 34 links wide; what is the value of it at 100*l.* per acre?
7. How many square feet of timber will make a chest 6 feet 6 inches long, 2 feet 3 inches wide, and 2 feet 6 inches deep?
8. A parallelogram is 14 feet 6 inches long, and 5 feet 3 inches between the sides; find its area in square feet.

Section 4.

9. A rectangular field is 4 chains 73 links long, and 5 chains 83 links wide; how many acres does it contain?
10. A rectangular field is 15 chains and 63 links long, and 12 chains 53 links wide; what is the rental of it at 3*l.* 15*s.* per acre?
11. A rectangular field is 8 chains 73 links long, and 10 chains *43 links* wide; what will be the price of mowing it at *9s. 6d.* per acre?

Section 5.

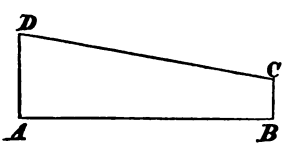
12. Find the square feet in the following four rectangular pieces of ground : 14 feet 6 inches long, and 5 feet 3 inches wide ; 10 feet 6 inches long, and 4 feet 4 inches wide ; 14 feet 6½ inches long, and 3 feet 5½ inches wide ; and 15 feet 6½ inches long, and 3 feet 4½ inches wide.

Section 6.

13. Find the contents of each of the three rectangular fields: 7.34 chains long, and 3.54 broad ; 3.14 chains long, and 2.24 broad ; and 5.16 chains long, and 3.89 broad.

Section 7.

To find the area of *A D B C*; add the heights *A D* and *B C*, and divide by 2 for a mean height; multiply the mean height by the length *A B*, and the result will be the area.



14. How many acres are there in a field, whose length is 13.14 chains, and perpendicular breadths at each extremity 132 yards, and 78 yards respectively.

15. Find the value of a piece of timber, whose dimensions are 14 feet 7 inches long, 3 feet 4 inches at one end, and 1 foot 4½ inches at the other, at 2*s.* 7½*d.* per square foot.

16. Find the value of a field, whose length is 33.74 chains, and perpendicular breadths at each extremity are 3.42 chains, and 2.23 chains respectively, at 57*l.* 17*s.* 6*d.* per acre.

Section 8.

17. The area of a field is 1 acre, whose length is 11½ chains, and breadth at one end ¼ chain ; what is the breadth at the other end ?

18. A piece of timber contains 30 square feet ; find the length, the perpendiculars at each end measuring 14 inches and 18 inches respectively.

19. A garden, which contains a rood, has its length equal to its greatest perpendicular length at one extremity, and the perpendicular at the other extremity is half the former ; find the length.

Problem 4.

To find the area of a circle, ellipse, and parabola.

Circle.

Rule I.—Square the diameter and multiply the result by .7854 for the area.

Rule II.—Square one fourth of the circumference, and divide the result by .7854 for the area.

Rule III.—Take half the circumference and half the diameter, and multiply them together for the area.

Rule IV.—Square the radius, and multiply the result by 3.1416 for the area.

Ellipse.

Rule.—Multiply the diameters and .7854 together for the area.

Parabola.

Rule.—Multiply the abscissa by the double ordinate; $\frac{2}{3}$ of this product will be the area.

Formulæ.

Let D = diameter of circle, c = its circumference, and r = its radius.

a, b = the diameters of the ellipse.

x, y = abscissa and double ordinate of parabola.

Circle.

$$\text{Area} = .7854\,D^2 = \left(\frac{c}{4}\right)^2 \div .7854 = \frac{c\,D}{4} = 3.1416r^2.$$

Ellipse.

$$\text{Area} = .7854\,a\,b.$$

Parabola.

$$\text{Area} = \frac{2\,x\,y}{3}.$$

It is readily seen that a circle is a polygon of an infinite number of sides, the circumference being the perimeter, and the radius the perpendicular. Hence, the area of a circle is the area of a triangle whose base is the circumference, and perpendicular height is the radius.

The squaring of a circle is a very ancient problem, the object of which is to find a square, geometrically, equal in area to a circle.

This problem, in consequence of its great difficulty, has engaged the attention of thousands in every age since the Greek epoch without the slightest success; but the labours of Van Ceulen, Sharp, and Rutherford have approximated to the solution so near, that the exact solution would not be attended with any practical benefit.

Example 12.—Section 7.

$$\text{Area of piston} = \frac{\pi D^2}{4} = \frac{3.1416 \times 56^2}{4} = 2463.0144 \text{ sq. in.}$$

$$\text{Pressure on the piston} = \frac{2463.0144 \times 10.5}{2240} = 11.545 \text{ tons.}$$

EXAMPLES.

Section 1.

1. Find the area of a circle, whose diameter is 15 inches.
2. Find the area of a circle, whose diameter is $8\frac{1}{2}$ inches.
3. Find the area of a circle, whose diameter is 6 feet $7\frac{3}{4}$ inches.
4. Find the area and circumference of a circle, whose diameter is $15\frac{3}{4}$ inches.

Section 2.

5. Find the area of a circle, whose circumference is 29.8452 inches.
6. What is the diameter and area of a circle, whose circumference is 251.328 inches?
7. What is the area and circumference of a circle, whose diameter is 7 feet 4 inches?

Section 3.

8. Find the areas and circumferences of the circles, whose diameters are 8 feet, 9 feet, 15 feet, $6\frac{1}{2}$ feet, and $24\frac{1}{2}$ feet.

Section 4.

9. Find the areas and circumferences of the circles, whose diameters are 12 feet, $13\frac{1}{4}$ feet, $16\frac{1}{4}$ feet, $21\frac{1}{8}$ feet, and 34 feet.

Section 5.

10. Find the areas and circumferences of the circles, whose diameters are 27 feet, 48 feet, 35 feet, 17¾ feet, and 26⅝ feet.

Section 6.

11. Find the areas and circumferences of the circles, whose diameters are 25 feet, 45 feet, 49 feet, 27 feet 5 inches, and 37 feet 7 inches.

Section 7.

12. The diameter of a piston for a steam-engine cylinder is 4 feet 8 inches, and the mean pressure of steam upon it is 10½ lbs. per square inch; required the pressure, in tons, on the piston.

13. The mean pressure of steam in a cylinder is 16 lbs. per inch, and the total pressure was observed to be 6 tons; required the diameter of the piston.

14. The area of a circle is 20.129 square feet; find the circumference and diameter.

Section 8.

15. Find the area of the ellipses, whose diameters are 16 feet, 4 feet; 26 feet, 9 feet; 25½ feet, 14¼ feet; 32⅛ feet, and 22¾ feet.

Section 9.

16. Find the area of an elliptical field, whose axes are 27 and 3 chains.

17. The area of an elliptical garden is 84 square yards, and one of its axes is 18 yards; required the other.

18. What is the diameter of a circular field, and also the side of a square field, each of which contains ½ an acre?

Section 10.

19. Find the area of a parabola, whose abscissa is 8 feet, and the double ordinate 15 feet.

20. Find the area of the parabolas, whose abscissa and double *ordinates* are as follow: 21¾ feet, 32½ feet, 25⅛ feet, 24½ *feet;* 25⅜ feet, and 41¼ feet.

Section 11.

21. The equatorial and polar diameters of the earth are 7926 miles, and 7899 miles; find the difference between a section passing through the equator, and another passing through a meridian.

22. Find the area of a parabola, whose abscissa and double ordinate are $52\frac{1}{4}$ feet, and $36\frac{1}{8}$ feet.

Section 12. (Requires algebra.)

23. The area of a circle is equal to 6 times its circumference; find its area and diameter.

24. The area of a circle is equal to the circumference and radius; find its area.

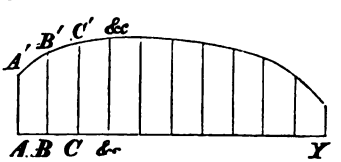
25. If 4 be added to the circumference of a circle, the sum will be the area; find its area.

26. The area of a circle is equal to 5 times its radius added to the circumference of a circle whose radius is 2; find the area.

27. The area of a circle is equal to its circumference, and area of an equilateral triangle inscribed within it; find the area of the circle and triangle.

PROBLEM 5.

To find the area of any curve by Simpson's Rule.



Divide $A\ Y$ into an even number of equal parts in the points B, C, *&c.*, then draw the ordinates $A\ A'$, $B\ B'$, $C\ C'$, *&c.* perpendicular to $A\ Y$, and meeting the curve in the points A', B', C', *&c.* The number of equal parts into which $A\ Y$ is divided must depend entirely upon the degree of curvature of the curve, and the accuracy with which the area is required. Generally, the greater the number of parts into which $A\ Y$ is divided the greater will be the accuracy of the area.

Rule.—Take half the sum of the first and last ordinates for the *first* sum.

Take twice the sum of all the *even* ordinates, for the *second* sum.

Take the sum of all the *uneven* ordinates, omitting the *first and last* ordinates, for a *third* sum.

Add the *first*, *second*, and *third* sums together, and multiply this sum by $\frac{1}{3}$ the common interval $A\ B$, and the product will be the area of the curve.

FORMULÆ.

Put $A\ A' = a_1$, $B\ B' = a_2$, $C\ C' = a_3$, &c.

$A\ B = B\ C$ &c. $= n$.

$$\therefore \text{Area} = \frac{2n}{3}\left\{\frac{a_1 + \text{last ordinate}}{2} + 2(a_2 + a_4 + a_6 + \&c.) + (a_3 + a_5 + a_7 + \&c.)\right\}$$

Example 11.—Section 5.

5	9	12
8	14	20
—	20	20
2)13	18	16
—	11	—
6.5 = first sum.	—	68 = third sum.
	72	144 = second sum.
	2	6.5 = first sum.
	—	—
	144 = second sum.	218.5
		4 = $\frac{1}{3}$ of com. in.
		—
		874 = area.

EXAMPLES.

Section 1.

1. The ordinates of a curve are 1, 2, 4, 6, 8, 7, 5 feet, and the common interval 2 feet; find the area.

2. The ordinates of a curve are 2, 3, 5, 8, 10, 14, 13, 9, 6, 3, 1 feet, and the common interval 9 inches; find the area.

Section 2.

3. The ordinates of a curve are 0, $1\frac{1}{2}$, $3\frac{1}{4}$, 4, 6, 7, 8, 10, 7, 5, 3, 2, 1 feet respectively, and the common interval 1 foot 3 inches; find its area.

4. The ordinates of a curve are 2, 8, 11, 14, 16, 13, 14, 11, 8, 7, 5, 3, 0 feet respectively, and the common interval is 1 foot 9 inches; find its area.

Section 3.

5. The ordinates of a curve are 0, 2, $3\frac{1}{2}$, $5\frac{1}{4}$, 6, 7, $8\frac{3}{4}$, 9, $7\frac{1}{4}$, $6\frac{3}{4}$, 5, 3, $2\frac{1}{2}$, $1\frac{1}{2}$, 0 feet respectively; find the area when the common interval is 1 foot 3 inches.

6. The ordinates of a curve are 5, 21.9, 26.9, 28.15, 28.9, 29.4, 29.6, 29.7, 29.7, 29.7, 29.7, 29.7, 29.6, 29.5, 29.3, 28.95, 28.5, 27.75, 26.9, 25.9, 24.5 feet respectively; find the area when the common interval is 10 feet.

Section 4.

7. The ordinates of a curve are 3, 4, 5, 7, 6, 2, 1 feet, and common interval 2 feet; required the area.

8. The ordinates of a curve are 1, 2. 4, 7, 9, 6, 5, 3, 2 feet, and common interval 1 foot 6 inches; find its area.

9. The ordinates of a curve are 17.50, 380.74, 672.8, 790.06, 855.21, 894.53, 909.07, 919.72, 921.26, 922.8, 922.8, 916.65, 907.53, 892.25, 874.37, 850.13, 815.83, 761.6, 690.52, 569.95, 185.84 feet respectively; find the area when the common interval is 10 feet.

Section 5.

10. The ordinates of a curve are 3, 5, 9, 4, 2 feet; and common interval 3 feet; find its area.

11. The ordinates of a curve are 5, 9, 12, 14, 20, 20, 20, 18, 16, 11, 8 feet, and common interval 6 feet; find the area.

12. The ordinates of a curve are 6, 14.65, 19.4, 22.55, 24.6, 25.95, 26.8. 27.1, 27.3, 27.3, 27.3, 27.25, 27.2, 26.95, 26.5, 25.75, 24.6, 23.05, 21.1, 18.5, 15.4, 11.1, 3.9 feet, and the common interval is 9 feet; find its area.

Section 6.

13. The ordinate of a curve are 5, 21, 26, 28, 28, 29, 27, 26, 25 feet, and the common interval 10 feet; required the area.

4. The ordinates of a curve are 9, 10, 12, 12, 12, 11, 8 feet, and the common interval 3 feet; find the area.

15. The ordinates of a curve are 43.5, 266.28, 453.13, 587.34, 659.93, 709.65, 741.82, 754.06, 760.86, 760.86, 760.86, 759.51, 758.16, 748.63, 732.11, 708.08, 668.94, 618.16, *549.93, 456*.09, 339.27, 187.72, 18.5 feet, the common *interval is* 9 feet; required the area.

Section 7.

16. The ordinates of a curve are 3, 9, 10, 12, 14, 15, 15, 15, 13, 11, 9, 4, 0 feet ; required the area when the common interval is 2 feet.

17. The ordinates of a curve are 604.96, 7531.28, 13637.29, 17577.27, 19165.04, 20323.38, 20972.6, 21280.59, 21359.7, 21359.7, 21359.7, 21359.7, 21359.7, 21093.23, 20673.81, 20295.02, 19482.6, 18619.13, 17241.16, 15643.06, 13137.14, 9045.65, 902.5 feet ; find the area when the common interval is 9 feet.

Section 8.

18. The ordinates of a curve are 10.85, 21.65, 25, 26.4, 26.95, 27.25, 27.4, 27.5, 27.5, 27.5, 27.5, 27.5, 27.5, 27.35, 27.2, 27.1, 26.8, 26.5, 26.05, 25.5, 24.7, 23.15, 18.5 feet; find the area when the common interval is 9 feet.

19. The ordinates of a curve are 0, 4, 6, 7, 7, 7, 5, 3, 0 feet; find the area when the common interval is 2 feet 6 inches.

Section 9.

20. The ordinates of a curve are 3.2, 13, 18.4, 21.75, 23.9, 25.15, 26, 26.5, 26.8, 26.9, 26.9, 26.7, 26.3, 25.7, 24.9, 23.65, 22, 19.6, 16.5, 11.9, 3.5 feet; required the area when the common interval is 10 feet.

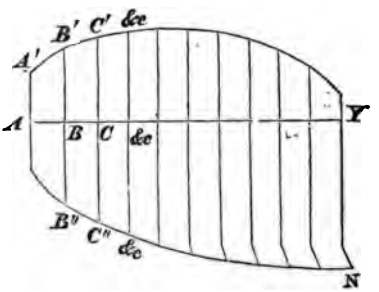
21. The ordinates of a curve are 0, 2, 4, 7, 8, 12, 14, 17, 16, 13, 11, 9, 7, 6, 4 feet, and the common interval 6 feet; required the area.

22. The ordinates of a curve are 0, 1, 3, 4, 5, 6, 7, 8, 9, 4, 2, 1, 0 feet, and the common interval 8 feet ; find its area.

PROBLEM 6.

To find the centre of gravity of any curve by Simpson's Rule.

Divide $A\ Y$ as in Problem 5.



Rule.—Find the area of $A\,A'\,Y$ by Problem 5.

Multiply the ordinates commencing at A, by the consecutive numbers 0, 1, 2, 3, 4, 5 &c. respectively. Let each of these products be represented by 0 at A, $B\,B''$ at B, $C\,C''$ at C, &c., the ordinates of the curve $A\,N$.

Find the area of the curve A N by Simpson's Rule.

Multiply the area of the curve $A\,N$ by the common interval, and divide this product by the area of the curve $A'\,Y$; the quotient will be the distance of the centre of gravity from the line $A\,A'$.

Rule 2.—Square each ordinate, and find the area of a curve, by Simpson's Rule, having these squares for the ordinates. Divide this area by the area $A'\,Y$, and half the quotient will be the distance of the centre of gravity from the line $A\,Y$.

FORMULÆ.

Put $A\,A' = a_1$; $B\,B' = a_2$; $C\,C' = a_3$, &c., and $A\,B = B\,C$, $= n$.

Put X = distance of centre of gravity from $A\,A'$.

Put Y = distance of centre of gravity from $A\,Y$.

$$\therefore X = n \left\{ \frac{\frac{2n}{3}\left\{\frac{0 + \text{last ordinate}}{2}\right.}{\frac{2n}{3}\left\{\frac{a_1 + \text{last ordinate}}{2}\right.} \right.$$

$$\left. \begin{array}{l} + 2\,(a_2 + 3a_4 + 5a_6 + \&c.) + 2a_3 + 4a_5 + 6a_7 + \&c. \\ \hline + 2\,(a_2 + a_4 + \&c.) + a_3 + a_5 + a_7 + \&c. \end{array} \right\} \Bigg\}$$

$$Y = \frac{\frac{2n}{3}\left\{\frac{a_1^2 + \text{last ordinate}}{2}\right.}{\frac{4n}{3}\left\{\frac{a_1 + \text{last ordinate}}{2}\right.}$$

$$\left. \begin{array}{l} + 2\,(a_2^2 + a_4^2 + a_6^2 + \&c.) + a_3^2 + a_5^2 + a_6^2 + \&c. \\ \hline + 2\,(a_2 + a_4 + a_6\ \&c.) + a_3 + a_5 + a_7 + \&c. \end{array} \right\}$$

Example 7.—Section 4.

Column 1. Ordinates of the curve.	Column 2. Contains the natural numbers from 0 to the number of ordinates.	Column 3. The product of Col. 1 by Col. 2.	Column 4. The squares of Col. 1.
1	0	0	1
2	1	2	4
4	2	8	16
7	3	21	49
9	4	36	81
6	5	30	36
5	6	30	25
3	7	21	9
2	8	16	4
55·5		230	320·5

The common interval is 1 foot 6 inches.

The respective columns put in Simpson's rule.

$$X = \frac{1\frac{1}{2} \times 230}{55{\cdot}5} = 6{\cdot}216$$

$$Y = \frac{\frac{1}{3} \times 320{\cdot}5}{55{\cdot}5} = 2{\cdot}887$$

EXAMPLES.

Section 1.

1. Find the centre of gravity of a curve, whose ordinates are 3, 4, 5, 7, 6, 2, 1 feet, and common difference 2 feet.

2. Find the centre of gravity of the curve in Example 1, Prob. 5.

Section 2.

3. *Find the* centre of gravity of the curve in Example 2, Prob. 5.

4. *Find the* centre of gravity of the curve in Example 3, Prob. 5.

Section 3.

5. Find the centre of gravity of the curve in Example 4, Prob. 5.

6. Find the centre of gravity of the curve in Example 5, Prob. 5.

Section 4.

7. Find the centre of gravity of the curve in Example 8, Prob. 5.

8. Find the centre of gravity of the curve in Example 10, Prob. 5.

Section 5.

9. Find the centre of gravity of the curve in Example 11, Prob. 5.

10. Find the centre of gravity of the curve in Example 13, Prob. 5.

Section 6.

11. Find the centre of gravity of the curve in Example 14, Prob. 5.

12. Find the centre of gravity of the curve in Example 16, Prob. 5.

Section 7.

13. Find the centre of gravity of the curve in Example 19, Prob. 5.

14. Find the centre of gravity of the curve, whose ordinates are 0, 1, 3, 4, 5, 6, 7, 8, 9, 4, 2, 1, 0 feet, and common interval 8 feet.

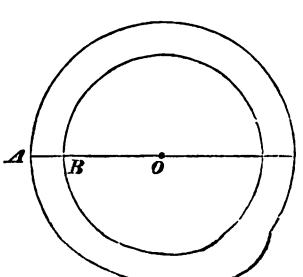
15. Find the centre of gravity of the curve, whose ordinates are 0, 2, 4, 7, 8, 12, 14, 17, 16, 13, 11, 9, 7, 6, 4 feet, and common interval 6 feet.

PROBLEM 7.

To find the area between two concentric circles.

Rule 1.—Find the area of a circle whose radius is $O\,B$, and also the area of the circle whose radius is $O\,A$. The difference between these areas is the concentric area.

Rule 2.—Multiply the sum and difference of the diameters by ·7854 for the area between the concentric circles.



FORMULA.

Let D = external diameter, and d = internal diameter.

$$\therefore \text{Area} = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}(D + d)(D - d).$$

Example 33.—Section 10.

$$\text{Area} = \frac{\pi}{4}(D + d)(D - d) = \frac{3{\cdot}1416}{4} \times (6 \text{ ft. } 6\tfrac{1}{2} \text{ in.}) \times 1\tfrac{7}{8} \text{ in.} = 115{\cdot}044 \text{ sq. in.}$$

Tensile strength $= 115{\cdot}044 \times 25\frac{1}{2} = 2933{\cdot}6$ tons.

Find the concentric area of the circles, whose external and internal diameters are as follow.

	External diameter.		Internal diameter.			External diameter.		Internal diameter.	
				Section 1.					
	ft.	in.	ft.	in.		ft.	in.	ft.	in.
(1)	7	0	5	0	(2)	12	0	10	0
(3)	11	9	7	6	(4)	7	10	4	6
				Section 2.					
(5)	10	9	8	4	(6)	18	4	10	4
(7)	19	0	18	0	(8)	14	5	5	7
				Section 3.					
(9)	20	4	18	7	(10)	15	6	14	9
(11)	20	8	16	7	(12)	33	3	30	4
				Section 4.					
(13)	25	7	24	10	(14)	15	7	14	5
(15)	26	5	23	1	(16)	13	7	10	7
				Section 5.					
(17)	15	4	11	11	(18)	5	4	3	4
(19)	5	6	2	1	(20)	3	4	2	4
				Section 6.					
(21)	24	8	20	7	(22)	31	4	29	5
(23)	27	7	24	8	(24)	27	10	22	11
				Section 7.					
(25)	*29*	1	20	3	(26)	24	8	19	9
(27)	*2*	3	1	3	(28)	1	6	1	0

The strength of materials to resist either compression or extension is proportional to the area of the section of the material; therefore, multiply the area in inches by the strength of one square inch (as per Table, page 63).

Section 8.

29. Find the tensile strength of a cast-iron cylinder, whose external and internal diameters are 6 feet 8 inches and 6 feet 6 inches.

30. Find the tensile strength of a cylinder of lignum-vitæ, whose external and internal diameters are 1 foot 8 inches and 10 inches.

Section 9.

31. What will a cylinder of oak bear in the direction of its length, the external and internal diameters being 2 feet 3 inches and 1 foot 8 inches.

32. Find the tensile strength of a cylinder of wrought iron, whose external and internal diameters are 1 foot 4 inches and 1 foot 3 inches.

Section 10.

33. Find the tensile strength of a cylinder of wrought iron, whose external and internal diameters are 3 feet 4 inches and 3 feet $2\frac{1}{8}$ inches.

34. What is the strength of a cast-iron cylinder, whose external and internal diameters are 2 feet 9 inches and 2 feet $7\frac{1}{4}$ inches?

Section 11.

35. What is the strength of a cylinder of brass, whose external and internal diameters are $5\frac{1}{2}$ and $5\frac{1}{4}$ inches?

36. What is the strength of a cylinder of copper, whose external and internal diameters are $4\frac{3}{8}$ and $4\frac{1}{8}$ inches?

Section 12.

37. Divide a circle, whose diameter is 8 feet, into two equal parts, by means of a concentric circle.

38. *Divide a circle, whose diameter is 9 feet, into three equal parts, by means of two concentric circles.*

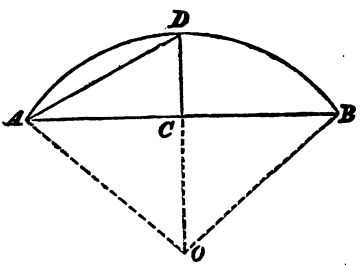
PROBLEM 8.

To find the area of a sector and segment of a circle.

Sector A O B D.

Rule.—Find the circumference of the segment *A D B*, by Problem 7, mensuration of lines. Then find the radius *O A*, by Problem 4, mensuration of lines.

One-half the product of the radius, and the arc *A D B*, will be the area of the sector *A O B D*.



Segment A C B D.

Rule.—The square root of the difference between the squares of *A O* and *A C* will be the line *O C*.

Then, the area of the triangle *A O B* is equal to the product of *O C* and *A C*.

The area of the segment is equal to the area of the sector, minus the area of the triangle *A O B*.

Newton's Approximate Rules.

1st. $\frac{4}{10}\left(AB + \frac{4\,AD}{3}\right) \times CD =$ area of segment, nearly.

2nd. $\frac{2\,AB \times CD}{3} + \frac{CD^2}{2\,AB} =$ area of segment, nearly.

3rd. $\left\{ 2\sqrt{AB^2 + CD^2} + \sqrt{AC^2 + CD^2} \right\} \times \frac{4\,CD}{15}$

4th. $\left\{ 7\sqrt{AC^2 + \frac{2\,CD^2}{7}} + \frac{4}{3}\sqrt{AC^2 + CD^2} \right\} \times$

$\frac{4\,CD}{25}$ very nearly.

Examples.

Section 1.

1. Find the area of the two segments of a circle, whose bases are 40 and 35 feet, and heights 4 and 5 feet.

Section 2.

2. Find the area of the three segments of circles, whose bases are 78, 240, and 90 feet, and heights $11\frac{1}{2}$, 24, and 30 feet.

Section 3.

3. Find the area of the three segments of circles, whose bases are 114, 153, and 194 feet, and height $10\frac{1}{2}$, $11\frac{1}{4}$, and $18\frac{3}{8}$ feet.

Problem 9.

To find the radii of the inscribed, circumscribed circles, and the area of the regular polygon, whose side is given.

No. of sides.	Name of Polygon.	Angle at the centre subtended by the side of the polygon.	Angle between the sides of the polygon.	Multiplier for the the radius of the inscribed circle.	Multiplier for the radius of the circumscribed circle.	Multiplier for the area of the polygon.
3	Trigon . .	120°	60	·2887	·57735	.433012
4	Tetragon .	90	90	·5000	·70711	1·000000
5	Pentagon .	72	108	·6882	·85065	1·720477
6	Hexagon .	60	120	·8660	1·00000	2·598076
7	Heptagon .	$51\frac{3}{7}$	$128\frac{4}{7}$	1·0383	1·15238	3·633912
8	Octagon .	45	135	1·2071	1·30656	4·828427
9	Nonagon .	40	140	1·3737	1·46190	6·181824
10	Decagon .	36	144	1·5388	1·61803	7·694209
11	Undecagon	$32\frac{8}{11}$	$147\frac{3}{11}$	1·7028	1·77473	9·365640
12	*Dodecagon*	30	150	1·8660	1·93185	11·196152

Rules.

1st. Multiply the side of the polygon by the tabular number opposite to it in the columns for radius of the inscribed and circumscribed circles respectively, and the product will be the radius of the inscribed and circumscribed circles respectively.

2nd. Multiply the square of the side of the polygon by the tabular number opposite to it in the column for the area, and the product will be the area.

Example.

Find the radii of the inscribed and circumscribed circles and area of an octagon whose side is 5 feet.

Radius of the inscribed circle $= 5 \times 1{\cdot}2071 = 6{\cdot}0355$ feet.

Radius of the circumscribed circle $= 5 \times 1{\cdot}30656 = 6{\cdot}53280$ ft.

Area of the octagon $= 25 \times 4{\cdot}828427 = 120{\cdot}71$ square feet.

Examples.

Section 1.

1. Find the radii of the inscribed and circumscribed circles of an undecagon, whose side is 4 feet 6 inches, and also its area.
2. Find the area, radii of the inscribed and circumscribed circles, of a pentagon, whose side is 5 feet 1½ inch.

Section 2.

3. Find the area, radii of the inscribed and circumscribed circles, of the base of a column of 7 sides, each of which is 7 feet 7½ inches.
4. Find the area, radii of the inscribed and circumscribed circles, of a hexagon, whose side is 12 feet 4 inches.

Section 3.

5. Find the area, radii of the inscribed and circumscribed circles, of a nonagon, whose side is 8 feet 10 inches.
6. *Find the area,* radii of the inscribed and circumscribed circles, *of a dodecagon,* whose side is 7 feet 3 inches.

Section 4.

7. Find the area of an octagonal garden, whose side is 3·42 chains.

8. Find the area of a pentagonal field, whose side is 5·18 chains.

Section 5.

9. The area of a pentagon is an acre; find the side.

10. The area of an octagon is 100 square yards; find the side.

11. The area of a decagon is a square rood; find its side in feet.

Section 6.

12. Find the area of the pentagon, when the radii of the inscribed and circumscribed circles are equal to the area of the pentagon.

13. Find the area of the octagon, when the difference between the radii of the inscribed and circumscribed circles is equal to the area of the octagon.

14. Find the area of the decagon, when the area and radius of the inscribed circle are equal to 100.

PROBLEM 10.

To find the angle and surface of a screw propeller blade.

Rule.—Divide the pitch by the diameter, and opposite this ratio, in the following Table, and in the column marked $\left(\frac{\text{pitch}}{\text{diameter}}\right)$ there will be found the angle: and in the column marked δ, there will be found a number which, multiplied by the length and diameter, will give the area of both blades of the screw.

Find the angle and area of the screw of the *Termagant*, whose pitch is 18 feet, length 3 feet, and diameter 15·5 feet.

$\frac{18}{15\cdot5} = 1\cdot16129$, opposite to which is 20° 15′, the angle of the screw.

$\therefore 1\cdot76237 \times 3 \times 15\cdot5 = 81\cdot95$ square feet, the area of both blades.

If very great accuracy be required, then the proportional *parts of these* numbers must be taken.

Angle of Screw.	Pitch / Diameter.	The value of δ.	Angle of Screw.	Pitch / Diameter.	The value of δ.
5°	·27486	5·87383	·30	·46951	3·57698
·5	·27946	5·78159	·35	·47419	3·54556
·10	·28407	5·69236	·40	·47886	3·51478
·15	·28867	5·60601	·45	·48354	3·48461
·20	·29328	5·52239	·50	·48822	3·45503
·25	·29789	5·44138	·55	·49290	3·42599
·30	·30250	5·36287			
·35	·30711	5·28675	9°	·49758	3·39753
·40	·31173	5·21289	·5	·50226	3·36961
·45	·31634	5·14122	·10	·50695	3·34221
·50	·32096	5·07163	·15	·51164	3·31531
·55	·32558	5·00403	·20	·51633	3·28892
			·25	·52103	3·26301
6°	·33019	4·93835	·30	·52572	3·23758
·5	·33482	4·87449	·35	·53042	3·21260
·10	·33944	4·81240	·40	·53512	3·18806
·15	·34406	4·75199	·45	·53982	3·16395
·20	·34869	4·69321	·50	·54453	3·14027
·25	·35331	4·63598	·55	·54924	3·11701
·30	·35794	4·58025			
·35	·36257	4·52595	10°	·55395	3·09416
·40	·36720	4·47305	·5	·55866	3·07169
·45	·37183	4·42148	·10	·56338	3·04961
·50	·37646	4·37119	·15	·56809	3·02789
·55	·38110	4·32215	·20	·57281	3·00655
			·25	·57754	2·98556
7°	·38574	4·27430	·30	·58226	2·96491
·5	·39038	4·22760	·35	·58699	2·94458
·10	·39502	4·18201	·40	·59172	2·92449
·15	·39966	4·13749	·45	·59645	2·90497
·20	·40431	4·09402	·50	·60119	2·88564
·25	·40895	4·05154	·55	·60592	2·86660
·30	·41360	4·01003			
·35	·41825	3·96945	11°	·61066	2·84788
·40	·42290	3·92978	·5	·61541	2·82949
·45	·42755	3·89100	·10	·62015	2·81131
·50	·43221	3·85304	·15	·62490	2·79336
·55	·43686	3·81586	·20	·62965	2·77582
			·25	·63441	2·75852
8°	·44152	3·77958	·30	·63916	2·74145
·5	·44618	3·74402	·35	·64392	2·72465
·10	·45085	3·70921	·40	·64869	2·70809
·15	·45551	3·67512	·45	·65345	2·69178
·20	·46018	3·64172	·50	·65822	2·67572
·25	·46484	3·60883	·55	·66299	2·65988

Angle of Screw.	Pitch / Diameter.	The value of δ.	Angle of Screw.	Pitch / Diameter.	The value of δ.
12°	·66777	2·64428	·30	·87124	2·14753
·5	·67254	2·62891	·35	·87616	2·13858
·10	·67732	2·61376	·40	·88109	2·12973
·15	·68211	2·59882	·45	·88602	2·12101
·20	·68689	2·58410	·50	·89096	2·11232
·25	·69168	2·56958	·55	·89589	2·10376
·30	·69647	2·55526			
·35	·70127	2·54115	16°	·90084	2·09531
·40	·70607	2·52724	·5	·90578	2·08694
·45	·71087	2·51351	·10	·91074	2·07866
·50	·71568	2·49997	·15	·91569	2·07049
·55	·72048	2·48661	·20	·92065	2·06239
			·25	·92561	2·05438
13°	·72529	2·47344	·30	·93058	2·04647
·5	·73019	2·46044	·35	·93555	2·03863
·10	·73493	2·44761	·40	·94053	2·03089
·15	·73975	2·43496	.45	·94551	2·02323
·20	·74457	2·42248	·50	·95050	2·01564
·25	·74940	2·41016	·55	·95549	2·00814
·30	·75423	2·39800			
·35	·75906	2·38599	17°	·96048	2·00071
·40	·76390	2·37414	·5	·96548	1·99336
·45	·76874	2·36244	·10	·97048	1·98610
·50	·77359	2·35090	·15	·97549	1·97890
·55	·77844	2·33950	·20	·98050	1·97178
			·25	·98552	1·96474
14°	·78329	2·32824	·30	·99054	1·95777
·5	·78814	2·31712	·35	·99557	1·95087
·10	·79300	2·30615	·40	1·00059	1·94404
·15	·79786	2·29531	·45	1·00563	1·93728
·20	·80273	2·28461	·50	1·01067	1·93058
·25	·80760	2·27402	·55	1·01572	1·92396
·30	·81247	2·26358			
·35	·81735	2·25325	18°	1·02076	1·91740
·40	·82223	2·24307	·5	1·02582	1·91091
·45	·82711	2·23299	·10	1·03088	1·90448
·50	·83200	2·22304	·15	1·03594	1·89811
·55	·83689	2·21321	·20	1·04101	1·89181
			·25	1·04608	1·88557
15°	·84179	2·20349	·30	1·05116	1·87939
·5	·84669	2·19391	·35	1·05624	1·87327
·10	·85159	2·18440	·40	1·06133	1·86721
·15	·85650	2·17502	·45	1·06643	1·86121
·20	·86141	2·16575	·50	1·07154	1·85527
·25	·86632	2·15659	·55	1·07663	1·84938

Angle of Screw.	Pitch / Diameter.	The value of δ.	Angle of Screw.	Pitch / Diameter.	The v of δ
19°	1·08174	1·84355	·30	1·30129	1·641
·5	1·08685	1·83778	·35	1·30664	1·637
·10	1·09196	1·83206	·40	1·31201	1·633
·15	1·09709	1·82639	·45	1·31738	1·629
·20	1·10222	1·82078	·50	1·32276	1·625
·25	1·10736	1·81522	·55	1·32814	1·621
·30	1·11249	1·80971			
·35	1·11764	1·80426	23°	1·33353	1·617
·40	1·12279	1·79885	·5	1·33892	1·613
·45	1·12795	1·79349	·10	1·34432	1·610
·50	1·13311	1·78819	·15	1·34973	1·606
·55	1·13827	1·78293	·20	1·35515	1·602
			·25	1·36057	1·598
20°	1·14344	1·77772	·30	1·36600	1·595
·5	1·14862	1·77255	·35	1·37144	1·591
·10	1·15381	1·76744	·40	1·37688	1·588
·15	1·15900	1·76257	·45	1·38233	1·584
·20	1·16419	1·75734	·50	1·38779	1·581
·25	1·16939	1·75236	·55	1·39326	1·577
·30	1·17459	1·74743			
·35	1·17980	1·74254	24°	1·39873	1·574
·40	1·18502	1·73769	·5	1·40421	1·570
·45	1·19024	1·73289	·10	1·40969	1·567
·50	1·19547	1·72813	·15	1·41518	1·563
·55	1·20070	1·72340	·20	1·42068	1·560
			·25	1·42619	1·557
21°	1·20594	1·71873	·30	1·43170	1·553
·5	1·21119	1·71409	·35	1·43722	1·550
·10	1·21644	1·70949	·40	1·44275	1·547
·15	1·22170	1·70492	·45	1·44829	1·543
·20	1·22696	1·70041	·50	1·45384	1·540
·25	1·23223	1·69593	·55	1·45939	1·537
·30	1·23750	1·69149			
·35	1·24278	1·68708	25°	1·46495	1·534
·40	1·24807	1·68271	·5	1·47052	1·531
·45	1·25337	1·67839	·10	1·47609	1·528
·50	1·25867	1·67409	·15	1·48167	1·524
·55	1·26397	1·66983	·20	1·48726	1·521
			·25	1·49286	1·518
22°	1·26928	1·66562	·30	1·49846	1·515
·5	1·27460	1·66143	·35	1·50407	1·512
·10	1·27993	1·65728	·40	1·50969	1·509
·15	1·28526	1·65316	·45	1·51532	1·506
·20	1·29060	1.64908	·50	1·52096	1·503
·25	1·29594	1·64503	·55	1·52660	1·50

Angle of Screw.	Pitch / Diameter.	The value of δ.	Angle of Screw.	Pitch / Diameter.	The value of δ.
26·	1·53225	1·49807	28·	1·67041	1·43430
·5	1·53791	1·49518	·5	1·67628	1·43185
·10	1·54358	1·49233	·10	1·68216	1·42944
·15	1·54926	1·48948	·15	1·68804	1·42704
·20	1·55494	1·48667	·20	1·69393	1·42465
·25	1·56064	1·48387	·25	1·69983	1·42229
·30	1·56634	1·48109	·30	1·70574	1·41994
·35	1·57205	1·47833	·35	1·71167	1·41760
·40	1·57777	1·47560	·40	1·71760	1·41528
·45	1·58349	1·47288	·45	1·72354	1·41279
·50	1·58922	1·47017	·50	1·72948	1·41068
·55	1·59497	1·46749	·55	1·73544	1·40842
27·	1·60072	1·46483	29·	1·74141	1·40615
·5	1·60648	1·46219	·5	1·74739	1·40390
·10	1·61225	1·45956	·10	1·75338	1·40168
·15	1·61803	1·45695	·15	1·75938	1·39946
·20	1·62381	1·45437	·20	1·76539	1·39726
·25	1·62960	1·45180	·25	1·77140	1·39508
·30	1·63541	1·44924	·30	1·77742	1·39290
·35	1·64122	1·44670	·35	1·78346	1·39074
·40	1·64704	1·44419	·40	1·78951	1·38860
·45	1·65287	1·44169	·45	1·79557	1·38648
·50	1·65871	1·43920	·50	1·80163	1·38436
·55	1·66456	1·43674	·55	1·80771	1·38225
			30·	1·81380	1·38016

Examples.

Section 1.

1. The screw of the *Ajax* is 16 feet diameter, 17 feet 11 inches pitch, and 3 feet 2 inches length; find the angle and area.

2. The screw of the *Arrogant* is 15 feet 6 inches diameter, 15 feet pitch, and 2 feet 6 inches length; find the angle and area.

3. *The screw of the Blenheim* is 16 feet diameter, 20 feet pitch, *and 3 feet* 4 inches length; find the angle and area.

Section 2.

4. The screw of the *Dauntless* is 14 feet 8 inches diameter, 18 feet pitch, and 3 feet length ; find the angle and area.

5. The screw of the *Desperate* is 13 feet diameter, 14 feet pitch, and 2 feet 4 inches length ; find the angle and area.

6. The screw of the *Dwarf* is 5 feet 8 inches diameter, 8 feet pitch, and 1 foot length ; find the angle and area.

Section 3.

7. The screw of the *Encounter* is 12 feet diameter, 15 feet pitch, and 2 feet 6 inches length ; find the angle and area.

8. The screw of the *Fairy* is 5 feet 4 inches diameter, 8 feet pitch, and 1 foot length ; find the angle and area.

9. The screw of the *Greenock* is 14 feet diameter, 13 feet pitch, and 2 feet 2 inches length ; find the angle and area.

Section 4.

10. The screw of the *Hogue* is 16 feet diameter, 20 feet pitch, and 3 feet 4 inches length ; find the angle and area.

11. The screw of the *Minx* is 4 feet 6 inches diameter, 5 feet pitch, and 1 foot length ; find the angle and area.

12. The screw of the *Niger* is 12 feet 6 inches diameter, 17 feet pitch, and 2 feet 10 inches length ; find the angle and area.

Section 5.

13. The screw of the *Phœnix* is 12 feet diameter, 12 feet 8 inches pitch, and 2 feet 1 inch length ; find the angle and area.

14. The screw of the *Plumper* is 9 feet diameter, 5 feet 7 inches pitch, and 1 foot length ; find the angle and area.

15. The screw of the *Rattler* is 10 feet diameter, 11 feet pitch, and 1 foot 3 inches length ; find the angle and area.

Section 6.

16. The screw of the *Reynard* is 8 feet 9 inches diameter, 8 feet pitch, and 1 foot 4 inches length ; find the angle and area.

17. The screw of the *Rifleman* is 8 feet diameter, 9 feet pitch, and 1 foot 6 inches length ; find the angle and area.

18. The screw of the *Sharpshooter* is 9 feet diameter, 9 feet pitch and 1 foot 6 inches length ; find the angle and area.

Section 7.

19. The screw of the *Teazer* is 4 feet 6 inches diameter, 5 feet 10 inches pitch, and 1 foot length ; find the angle and area.

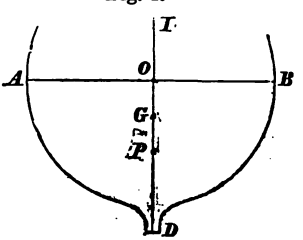
20. The screw of the *Vulcan* is 14 feet diameter, 16 feet 6 inches pitch, and 2 feet 9 inches length ; find the angle and area.

21. The screw of the *Conflict* is 13 feet 6 inches diameter, 16 feet 6 inches pitch, and 2 feet 9 inches length ; find the angle and area.

PROBLEM 2.

To find the metacentre of a floating body.

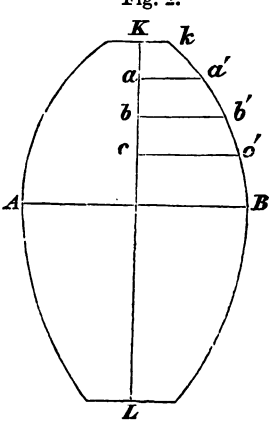
Fig. 1.



Let $A\ D\ B$ be a section of a floating body, divided symmetrically by the line $O\ D$, G its centre of gravity, and P the centre of gravity immersed, corresponding to the water-line $A\ B$.

The conditions of stability are, the weight of water displaced is equal to the weight of the floating body, and the line joining P with G is perpendicular to the water-line $A\ B$.

Fig. 2.



Let Fig. 2 be the water-line section passing through $A\ B$, Fig. 1 ; and V the displacement in cubic ft.

Divide the line $K\ L$ into an even number of equal parts, each equal to (n), at the points a, b, c, &c. ; and draw the ordinates $a\ a'$, $b\ b'$, $c\ c'$, &c., perpendicular to the axis $K\ L$.

Cube each of the ordinates $K\ k$, $a\ a'$, $b\ b'$, &c., and put them in Simpson's Rule and call the result B.

Then, $$P\ I = \frac{2\ B}{3\ V}.$$

If G is below I, the floating body is stable; but if it is above, then the *floating body is unstable.* (See Fincham's Outlines of Ship-building, page 157.)

Examples.

1. Find the distance $P I$ in the *Albion*, whose tonnage is 4267 tons, and half ordinates of the water-line section are 3.5, 16.25, 23.20, 26.8, 28.45, 29.3, 29.65, 29.9, 29.95, 30, 30, 29.8, 29.6, 29.2, 28.8, 28.15, 27.05, 25.6, 23.25, 18.25, 4.6, and common interval 10 feet.

$$\text{The volume } V = \frac{2240 \times 4267}{64} = 149345 \text{ cubic feet.}$$

The divisor 64 is used for sea-water.

3.5	cubed =	42.87	29.8	cubed =	26463.60
16.25	,, =	4291.02	29.6	,, =	25934.34
23.2	,, =	12487.17	29.2	,, =	24897.09
26.8	,, =	19248.83	28.8	,, =	23887.87
28.45	,, =	23027.50	28.15	,, =	22306.69
29.3	,, =	25153.76	27.05	,, =	19792.55
29.65	,, =	26065.98	25.6	,, =	16777.22
29.9	,, =	26730.90	23.25	,, =	12568.08
29.95	,, =	26865.22	18.25	,, =	6078.39
30.	,, =	27000.00	4.6	,, =	97.34
30.	,, =	27000.00			

42.87 first term.
97.34 last term.

2)140.21

70.10

Even terms.	Odd terms.
4291.02	12487.17
19248.83	23027.50
25153.76	26065.98
26730.90	26865.22
27000.00	27000.00
26463.60	25934.34
24897.09	23887.87
22306.69	19792.55
16777.22	12568.08
6078.39	
198947.50	197628.71
2	
397895.00	
197628.71	
70.10	
595593.81	
20	
3)11911876.20	
3970625.4 = B.	

$$\text{Then, } P I = \frac{2B}{3V} = \frac{2 \times 3970625 \cdot 4}{3 \times 149345} = 17 \cdot 72 \text{ feet.}$$

2. Find the distance PI in the *Rodney*, whose tonnage is 4267 tons, and whose half ordinates at the water-line section are 7.25, 17.8, 22.6, 25.1, 25.9, 26.45, 26.8, 26.95, 27, 27, 27, 27, 27, 26.9, 26.7, 26.5, 26.1, 25.65, 24.85, 23.7, 21.5, 17.35, 5, and common interval 10 feet.

3. Find the metacentre in a vessel whose displacement is 16397 cubic feet, and whose half ordinates at the water-line section are 3.95, 8.85, 12.2, 14.05, 15.05, 15.6, 15.8, 15.8, 15.7, 15.65, 15.35, 14.9, 14.2, 13.15, 11.1, 7.1, 1.15, and common interval 6 feet.

4. Find the metacentre of a vessel whose displacement is 15034 cubic feet, and whose half ordinates at the water-line section are 2.2, 6.35, 9.7, 12.2, 13.85, 14.8, 15.5, 15.7, 15.85, 15.6, 15.35, 14.8, 14.05, 12.85, 10.8, 7.15, 2.65, and common interval 6 feet.

5. Find the metacentre of a vessel whose displacement is 148914 cubic feet, and whose half ordinates at the water-line section are 6.9, 16.8, 21.8, 24.5, 25.78, 26.35, 26.75, 27, 27, 27, 27, 27, 27, 27, 27, 26.95, 26.7. 26.3, 26.05, 25.5, 24.6, 23.45, 21.35, 17.85, 8.1, and common interval 8 feet 4 inches.

If the water-line section be a parallelogram, whose length and breadth are l and b, then $B = \frac{b^3 l}{8}$.

If the water-line section be a circle, whose radius is r, then $B = \frac{3\pi r^4}{8}$.

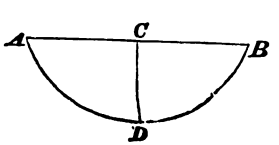
If the water-line section be an ellipse, whose semiaxes are a and b, then $B = \frac{3\pi a b^3}{8}$.

If the water-line section be a parabola, whose abscissa and corresponding ordinate are x and y, then $B = \frac{2xy^3}{5}$.

Miscellaneous Formulæ.

Cycloid.—The area of the cycloid is three times the area of the generating circle.

Catenary.—The area $ADBA = CD.AB - t(\overset{\frown}{ADB} - AB)$

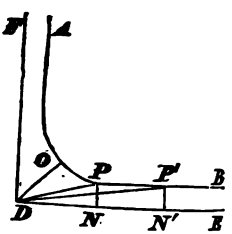


But, $\widehat{A\,D\,B} = t\left(E^{\frac{B\,C}{t}} - E^{-\frac{B\,C}{t}}\right)$, where $E = 2{\cdot}71828$, and t = the tension at D.

Involute of a Circle.—Let $A\,P$ be the arc of the involute, and O the centre of the circle.

The area included, by the radius $O\,A$, $A\,P$, and $O\,P$ = the cube of the string unwound, divided by three times the diameter of the circle.

Rectangular Hyperbola.—The lines, $F\,D$, $D\,E$ are the asymptotes, and the area included by two ordinates $P\,N$ and $P'\,N'$ is equal to $\frac{D\,O^2}{2}$ log. $\left(\frac{D\,N'}{D\,N}\right)$ = area of $P\,D\,P'$.



Spiral of Archimedes.—Let R = the radius vector where the angle is $2\,\pi$, and r any other radius vector.

$$\text{Area} = \frac{\pi\,r^3}{3\,R}.$$

Evolute of an Ellipse.—The whole area = $\frac{3\,\pi\,(a^2 - b^2)^2}{8\,a\,b}$,

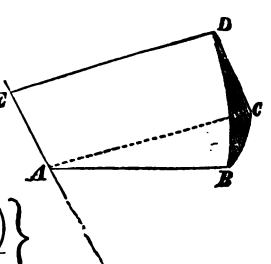
where a and b are the semi-axes of the ellipse.

Screw-propeller blade.—

Put $h = A\,E$, the length of screw.

$r = A\,B$, the radius of screw.

$A = \angle\ D\,B\,C$ of the helix $B\,D$.



Area of blade $A\,E\,B\,D\,A$ =

$$h\,r\left\{\frac{\text{Cosec. } A + \tan. A \log.\left(\text{Cot.}\frac{A}{2}\right)}{2}\right\}$$

MECHANICAL PROPERTIES OF THE MATERIALS OF CONSTRUCTION.

Moseley's Engineering and Architecture.

Name of Material.	Specific Gravity.	Weight of one cubic foot in pounds avoir-dupois.	Force in pounds avoir-dupois per square inch to pull it asunder.	Force per square inch in pounds to crush it.
Atmospheric air .		·0768		
Alder	·8	50·00	14186	6895
Ash	from ·69 to ·845	from 43·12 to 53·81	17207	9023
Beech	from ·854 to ·69	from 53·37 to 43·12	16817	8548
Birch	·792	49·5	1500	5467
Box	·96	60·0	19891	10299
Brass (cast) . . .	8·399	525·0	17968	10304
Copper (cast). . .	8·607	537·93	19072	
Deal (Memel) . . .	·59	36·87		
„ (Christiania) .	·689	43·62	12400	
„ (Norway spruce)	·34	21·25	17600	
„ (English) . .	·47	29·37	7000	
Elder	·695	43·43	10230	8467
Elm (seasoned) . .	·588	36·75	13489	10331
Fir (New England).	·553	34·56		
„ (Riga)	·753	47·06	12203	6167
Granite	2·625	164·00		
Iron (wrought) . .	7·7	481·2	25½ tons	
„ (cast, cold blast)	7·066	441·62	16683	106375
„ (cast, hot blast)	7·046	440·37	13505	108540
Larch	·522	32·62	10220	3201
Lead (cast, English)	11·446	717·45	1824	
Lignum vitæ . . .	1·22	76·25	11800	
Mahogany (Spanish)	·8	50·0	16500	
Oak (English) . .	·934	58·37	17300	4684 9509
Pine (red)	·657	41·06		5375
„ (yellow). . .	·461	28·81		5445
Steel (soft) . . .	7·78	486·25	120000	
Teak (dry). . . .	6·57	41·06	15000	12101
Tin (cast)	7·291	455·68	5322	
Water (rain) . . .	1·000	62·5		

MENSURATION OF SOLIDS.

1728 cubic inches = one cubic foot.

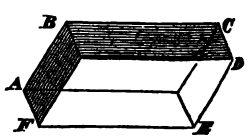
27 cubic feet = one cubic yard.

277.274 cubic inches = .16046 cubic feet = an imperial gallon.

Problem 1.

To find the volume of a parallelopiped.

"A parallelopiped is a solid figure contained by six quadrilateral figures, whereof every opposite two are parallel."—Euclid XI, Def. A.



Rule.—Multiply the area of the plane *A E* by the perpendicular distance between the parallel planes *A E* and *B D*.

When a body, such as a ship for instance, floats in water it displaces a certain quantity of water. The quantity of water thus displaced is equal in weight to the floating body. Hence, to find the displacement of a ship, it is only necessary to find the cubic feet of water, the weight of which is equal to the weight of the ship; this is done by dividing the weight of the ship in pounds by the weight of a cubic foot of water in pounds, and the quotient will be the displacement in cubic feet.

Example 13.—Section 5.

The weight = $30{\cdot}75 \times 2{\cdot}25 \times 1.5 \times 58{\cdot}37 = 6057{\cdot}711$ lbs.

Put x = the displacement in cubic feet.

$\therefore 62\frac{1}{2} \times x$ = the weight of water displaced.

$$\therefore x = \frac{6057{\cdot}711}{62\frac{1}{2}} = 96{\cdot}923.$$

Examples.

Section 1.

1. The length, breadth, and depth of a parallelopiped are 13, 4, and 2 feet; find its volume.
2. The length, breadth, and depth of a parallelopiped are 30 feet 6 inches, 2 feet 3 inches, and 3 feet 4 inches; find its volume.
3. A metal tank, whose length is 6 feet, and breadth 2 feet, holds 300 gallons; required its depth.

Section 2.

4. What weight of water will a tank hold, whose length, breadth, and depth are 40 yards, 30 yards, and 2 yards?
5. Find the volume and weight of a granite stone, whose length, breadth, and depth are 6 feet, 4 feet, and 3 feet 6 inches.
6. The inside dimensions of a wrought-iron tank are 3 feet 5 inches, 5 feet 7 inches, and 2 feet 3 inches, and the outside dimensions 3 feet 5¼ inches, 5 feet 7¼ inches, and 2 feet 3¼ inches; find the weight of the tank, and the gallons of water it will hold.

Section 3.

7. What is the weight of a balk of Riga fir, whose length, breadth, and depth are 25 feet, 2 feet 6 inches, and 1 foot 9 inches.
8. What is the weight of a balk of lignum vitæ, whose length, breadth, and thickness are 5 feet, 4 feet, and 3 feet.
9. What is the weight of a granite stone, whose length, breadth, and thickness are 9 feet 2 inches, 8 feet 3 inches, and 5 feet 6 inches?

Section 4.

10. A block of granite, whose dimensions are 8 feet by 5 feet 6 inches by 3 feet 2 inches, is suspended in water; what weight will it lose?
11. A balk of Riga fir, whose length, breadth, and thickness are 24 feet, 3 feet 3 inches, and 2 feet 6 inches, floats in water; how many cubic feet of water will it displace?
12. What will be the weight of a balk of oak, whose length, breadth, and depth are 25 feet 6 inches, 2 feet 4 inches and 1 foot 3 inches?

Section 5.

13. Find the displacement of a rectangular balk of oak, whose dimensions are 30 feet 9 inches, by 2 feet 3 inches, by 1 foot 6 inches.

14. Find the weight of a balk of teak, whose dimensions are 40 feet 8 inches, 9 inches, and 6 inches.

15. Find the displacement of a rectangular balk of Memel deal, whose dimensions are 35 feet 6 inches, 3 feet 6 inches, and 2 feet 4 inches.

Section 6.

16. A balk of red pine is 26 feet 6 inches long, 1 foot 3 inches wide, and 9 inches thick; what weight will it lose when immersed 13 feet 3 inches in the direction of its length?

17. Find the weight of a balk of lignum vitæ, whose dimensions are 16 feet, 1 foot 4 inches, and 1 foot 3 inches.

18. Find the weight of a balk of Spanish mahogany, whose dimensions are 15 feet 8 inches, 2 feet 1½ inch, and 3 feet 4½ inches.

Section 7.

19. Find the weight of a balk of elm, whose dimensions are 25 feet 3½ inches, 1 foot 3 inches, and 2 feet 4 inches.

20. Find the value of a balk of oak, whose dimensions are 30 feet 5 inches, 1 foot 7¼ inches, and 1 foot 3½ inches, at 3*s.* 6*d.* per cubic foot.

21. Find the value of a balk of Memel deal, whose dimensions are 35 feet 6 inches, 2 feet 1⅜ inch, and 1 foot 2⅜ inches, at 4*s.* 9*d.* per cubic foot.

Section 8.

22. What is the weight of a balk of larch, whose dimensions are 40 feet 7 inches, 3 feet 2½ inches, and 2 feet 1½ inch?

23. How many gallons of water will a cistern hold, whose length, breadth, and depth are 26 feet 10 inches, 3 feet 6½ inches, and 2 feet 1¼ inch?

24. Find the displacement of a balk of English oak, whose dimensions are 19 feet 11 inches, 1 foot 11 inches, and 1 foot 11 inches.

25. What weight will be lost by immersing a balk of yellow pine *10 feet* 2 inches in the direction of its length? the dimensions are 20 feet 4 inches, 2 feet 6½ inches, and 3 feet 1½ *inch?*

Section 9.

26. How many cubes of 2 inches square can be cut from a balk of timber 12 feet 8 inches, by 2 feet 5 inches, by 1 foot 7 inches?

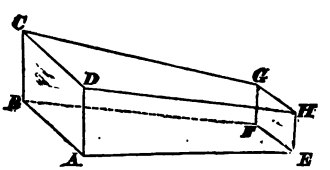
27. How many 4-inch cubes can be cut from a balk of timber 15 feet 4 inches long, by 2 feet 10 inches wide, and 1 foot 8 inches deep?

28. How many 3-inch cubes can be cut from a balk of timber 17 feet 9 inches, by 1 foot 9 inches, by 1 foot 3 inches?

PROBLEM 2.

To find the volume of a prismoid and wedge.

Let the parallelograms $A\ C$ and $E\ G$ be perpendicular to the plane $A\ F$, and $A\ B$ and $E\ F$ perpendicular to the length $A\ E$.



Rule.—To the area of the two ends ($A\ C$) and ($E\ G$) add half the parallelograms formed by the height ($A\ D$) of one end and the breadth ($E\ F$) of the other; multiply this sum by one-third of the length, and the product will be the volume.

FORMULÆ.

Let B, B' be the breadths $A\ B$ and $E\ F$.

And h, h' be the heights $A\ D$ and $E\ H$.

And L be the length $A\ E$.

$$V = \frac{L}{3}\left(B\,h + B'\,h' + \frac{B\,h' + B'\,h}{2}\right)$$

The prismoid becomes a wedge when h' is equal to nothing.

Then, $V = \frac{h\,L}{3}\left(B + \frac{B'}{2}\right)$ for the wedge.

Find the volume of a balk of timber, whose height and breadth at one end are 2 feet 6 inches and 2 feet 9 inches, and at the other end the height and breadth are 1 foot 3 inches and 1 foot 9 inches, and the length 15 feet 6 inches.

$$V = \frac{L}{3}\left(Bh + B'h' + \frac{Bh' + B'h}{2}\right)$$

$$= \frac{15\frac{1}{2}}{3}\left(2\frac{1}{2} \times 2\frac{3}{4} + 1\frac{1}{4} \times 1\frac{3}{4} + \frac{2\frac{1}{2} \times 1\frac{3}{4} + 1\frac{1}{4} \times 2\frac{3}{4}}{2}\right)$$

$$= \frac{31}{6}\left(\frac{5 \times 11}{8} + \frac{5 \times 7}{16} + \frac{5 \times 7}{16} + \frac{5 \times 11}{32}\right)$$

$$= \frac{31}{6}\left(\frac{55}{8} + \frac{35}{16} + \frac{35}{16} + \frac{55}{32}\right)$$

$$= 2144\tfrac{1}{8} \text{ cubic feet.}$$

Examples.

Section 1.

1. Find the volume of a stone, whose height and breadth are 3 feet 4 inches and 4 feet at one end, 2 feet 6 inches and 3 feet at the other, and whose length is $6\frac{1}{2}$ feet.
2. Find the volume of a balk of timber, whose height and breadth are 3 feet and 2 feet at one end, and 14 inches and 8 inches at the other, and length 15 feet.

Section 2.

3. Find the volume in cubic feet of a balk of timber, whose height and breadth are 15 inches and 10 inches at one end, and 10 inches and 6 inches at the other, and length 25 feet.
4. What is the price of a balk of timber, whose height and breadth are 7 inches and 12 inches at one end, and 5 inches and 9 inches at the other, and length 16 feet, at 3*s.* 6*d.* per cubic foot?

Section 3.

5. Find the weight of a prismoidal block of granite, whose height and breadth are 3 feet 3 inches and 2 feet 8 inches at one end, and 2 feet 9 inches and 1 foot 6 inches at the other, and length 4 feet 8 inches.
6. Find the weight of a prismoidal block of Riga fir, whose height and breadth are 16 inches and 14 inches at one end, and 10 inches and 8 inches at the other, and length 18 feet

Section 4.

7. Find the volume of a balk of timber, whose length is 27 feet 3 inches, breadth 2 feet 7 inches, and height at one end 13 inches, and at the other end 7 inches.

8. Find the volume of a balk of timber, whose length is 14 feet 8 inches, breadth 2 feet 4 inches, and thickness at one end 15 inches, and at the other end 23 inches.

9. Find the volume of a balk of timber, whose length is 27 feet 9 inches, breadth 2 feet 11 inches, and thickness at one end 19 inches, and at the other end 25 inches.

10. Find the volume of a balk of timber, whose length is 9 feet 5 inches, breadth 3 feet 1 inch, and thickness at one end 7 inches, and at the other end 15 inches.

Section 5.

11. Find the volume of a wedge, whose breadth and height are 1 foot 6 inches and 6 inches, and length of edge and length are 1 foot 9 inches and 2 feet.

12. What is the volume of a wedge, whose breadth and height are 3 feet and 8 inches respectively, length of edge and length are 1 foot 6 inches and 3 feet.

13. A wedge of oak is 2 feet 6 inches broad, and height 10½ inches, the length of edge is 1 foot 5 inches, and length 4 feet 9 inches; find its volume and weight.

PROBLEM 3.

Find the volume of a cylinder or prism, and concentric cylinder or prism.

Rule.—Multiply the area of the base of the cylinder or prism, by the height or length *C B*.

FORMULA.

Put D = diameter of base.

l = length of the cylinder.

V = its volume.

Then, $V = \frac{\pi D^2 l}{4}$

For a concentric cylinder, find the volume of the inte external cylinders, and their difference will be the c volume.

These rules will apply to any prism.

Question 18.—Section 7.

The external diameter is 1·25 feet, and the internal di $\frac{115}{96}$ feet.

$$\text{The weight in tons} = \frac{3\cdot1416}{4}\left\{\left(\frac{5}{4}\right)^2 - \left(\frac{115}{96}\right)^2\right.$$

$$\times \frac{3580 \times 441\cdot62}{2240} = 104\cdot2355 \text{ tons.}$$

$$\text{Then, } 104\cdot2355 \times 32\cdot75 = £3413\cdot7.$$

EXAMPLES.

Section 1.

1. The diameter and height of a cylinder are 3 feet an find its volume.

2. The diameter and height of a cylinder are 4 feet 6 in 8 feet 9 inches; find its volume.

3. The diameter and height of a cylinder are 5 feet 9 in 9 feet 8 inches; find its volume.

4. The diameter and height of a cylinder are 2 feet 6 in 24 feet 3 inches; find its volume.

Section 2.

5. The diameter and height of a cylinder are 2 feet 3 in 24 feet 8 inches; find its volume.

6. The diameter and length of a cylinder are 3 feet 4½ and 34 feet 9 inches; find its volume.

7. The external and internal diameters and length of a c *cylinder* are 3 feet 4 inches and 3 feet 3½ inches a *8 inches*; find its weight.

Section 3.

8. What is the weight of a cylinder of Riga fir, whose diameter and length are 3 feet 4 inches and 18 feet 9 inches?

9. What quantity of water will a cylinder of yellow pine displace, whose diameter and length are 2 feet 4 inches and 12 feet 2 inches?

10. Find the weight of a cylinder of oak, whose diameter and length are 2 feet $4\frac{3}{8}$ and 15 feet 8 inches.

Section 4.

11. Find the displacement of a cylinder of red pine, whose diameter and length are 3 feet 6 inches and 36 feet $9\frac{1}{2}$ inches.

12. What is the weight of a cylinder of lignum vitæ, whose diameter and length are 1 foot $3\frac{1}{2}$ inches and 5 feet 5 inches?

13. The circumference of a cylinder is 8 feet $4\frac{1}{2}$ inches, and length 32 feet 10 inches; find its volume.

Section 5.

14. Find the volume of a prism, whose section is a triangle having its three sides 3 feet 4 inches, 2 feet 6 inches, and 4 feet 6 inches, and its length 24 feet 6 inches.

15. Find the volume of a prism, whose section is a parabola, having 3 feet 9 inches for abscissa, and 6 feet 10 inches for double ordinate, and length 35 feet 9 inches.

Section 6.

16. Find the volume of a prism, whose section is an ellipse, and whose diameters are 10 feet 3 inches and 6 feet 4 inches, and length 27 feet 6 inches.

17. Find the weight of a wrought-iron cylinder, whose external and internal diameters, and length are 3 feet 6 inches and 3 feet $5\frac{1}{2}$ inches, and 6 feet.

Section 7.

18. Estimate the cost of a mile of gas-pipes at 32*l.* 15*s.* per ton, the external diameter being 15 inches, and thickness $\frac{5}{16}$ of an inch.

19. Find the price of 300 yards of brick-tunnel at 2*l.* 10*s.* per cubic yard, the internal diameter being 4 feet 6 inches and thickness 1 foot 6 inches.

Problem 4.

Find the volume and surface of a sphere or globe.

Rule.—Multiply the cube of the diameter by 3·1416, and the result by 6, and the quotient will be the volume.

Rule.—Multiply the square of the diameter by 3·1416, product will be the surface of the sphere.

Formulæ.

Put D = diameter, V = the volume, and S = the surf

$$\text{Then, } V = \frac{\pi D^3}{6} \text{ and } S = \pi D^2$$

Example 15.—Section 12.

$$\text{From the formula, } 94 = \frac{3\cdot1416 \times}{6} \quad \therefore D = \left(\frac{94\times6}{3\cdot141}\right.$$

$$= 5\cdot6412 \text{ feet.}$$

$$\text{Surface} = 3\cdot1416 \times D^2 = 3\cdot1416 \times (5\cdot6412)^2 = 99\cdot976 \text{ squa}$$

Put x the feet of water displaced.

Then, $62\cdot5 \times x = 94 \times 47\cdot06$, where $62\cdot5$, is the den water, and $47\cdot06$ is the density of Riga fir.

$$\therefore x = \frac{94 \times 47\cdot06}{62\cdot5} = 70\cdot7782 \text{ cubic feet.}$$

Examples.

Section 1.

1. Find the volume and surface of a sphere, whose diam 18 feet.
2. Find the volume and surface of a sphere, whose diam 24 feet.
3. Find the volume and surface of a sphere, whose diam 25 feet.

Section 2.

4. What is the volume and surface of a globe, whose di is 23½ feet?
5. *What is* the volume and surface of a globe, whose di *is 4 feet 6* inches?

Section 3.

6. Find the weight, surface, and volume of a globe of lignum vitæ, whose diameter is 8 feet 9 inches.

Section 4.

7. Find the surface, volume, and weight of a sphere of Riga fir, whose diameter is 10 feet 10 inches.

Section 5.

8. Find the surface, weight, and volume of a sphere of brass, whose diameter is 4 feet 5 inches.

Section 6.

9. Find the surface, volume, and displacement of a sphere of yellow pine, whose diameter is 8 feet 5 inches.

Section 7.

10. Find the weight, volume, and surface of a sphere of yellow pine, whose diameter is 5 feet 11 inches.

Section 8.

11. Find the surface, volume, weight, and displacement of a globe of English oak, whose diameter is 24 feet 7 inches.

Section 9.

12. Find the surface and volume of the spheres, whose diameters are as follow : 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 feet.

Section 10.

13. Find the diameter, weight, and surface of a globe of English oak, whose volume is 24 cubic feet.

Section 11.

14. Find the diameter, weight, and surface of a sphere of lignum vitæ, whose volume is 36 cubic feet.

Section 12.

15. Find the diameter, surface, and displacement of a sphere of Riga fir, whose volume is 94 cubic feet.

Section 13.

16. Find the diameter, surface, and weight of a cast-iron ball, whose volume is 84 cubic feet.

Section 14.

17. Find the diameter, surface, and displacement of a globe of oak, whose volume is 32 cubic feet.

Section 15.

18. Find the diameter, surface, and displacement of a sphere of yellow pine, whose volume is 42 cubic feet.

Section 16.

19. Find the diameters and volumes of two spheres, whose surfaces are 35 and 84 square yards.

Section 17.

20. Find the diameters and volumes of two spheres, whose surfaces are 94 and 55 square yards.

Section 18.

21. Find the diameters and volumes of two spheres, whose surfaces are 64 and 84 square yards.

Section 19.

22. Find the diameters of two cannon-balls, whose weights are 64 and 32 lbs.

Section 20.

23. Find the diameters of three cannon-balls, whose weights are 12, 18, and 24 lbs.

Section 21.

24. Find the diameters of three cannon-balls, whose weights are 42, 68, and 84 lbs.

Section 22.

25. The volume of a sphere is equal to the area of a circle having the same diameter as the sphere; find the diameter and volume.

26. The volume of a sphere is equal to the circumference of a circle having the same diameter as the sphere; find the diameter and volume.

27. The volume of a sphere is equal to the sum of the circumference and area of a circle having the same diameter as the sphere; find the diameter and volume.

Section 23.

28. The volume of a sphere is equal to its surface ; find the diameter and volume.

29. The volume of a sphere is equal to its surface added to the circumference of a circle having the same diameter as the sphere ; find its diameter, surface, and volume.

Section 24.

30. Find the volume and weight of a spherical shell of cast-iron, whose external and internal diameters are 8 and 5 inches respectively.

$$\frac{\pi D^3}{6} - \frac{\pi d^3}{6} = \frac{\pi}{6}(D^3 - d^3) = \frac{3{\cdot}1416}{6}(8^3 - 5^3) =$$

202·6332 cubic inches.

$$\text{Then, } \frac{202{\cdot}6332 \times 441}{1728} = 51{\cdot}7 \text{ lbs.}$$

31. Find the volume and weight of a spherical shell of wrought-iron, whose external and internal diameters are 9 and 6 inches respectively.

32. Find the volume and weight of a spherical shell of oak, whose diameters are 15 and 10 inches.

33. Find the volume and weight of a spherical shell of Riga fir, whose diameters are 13 and 4 inches.

Section 25.

34. The weight of a cast-iron shell is 42 lbs., and its external diameter 9 inches ; find its internal diameter.

35. Find the volume and weight of a spherical shell of copper, whose external and internal diameters are 1 foot 6 inches and 1 foot 3 inches.

Section 26.

36. How many gallons of water will fill a hemispherical shell of copper, whose external and internal diameters are 2 feet 6 inches and 2 feet 5⅛ inches ? and what is the value of the copper at 14*d.* per pound ?

37. The weight of a wrought-iron shell is 68 lbs., and its external diameter is 12 inches ; find its internal diameter.

Section 27.

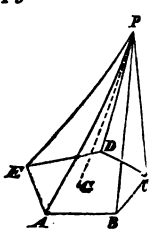
38. A hemispherical copper will hold 40 gallons; find its diameter.

39. The weight of a cast-iron shell is 112 lbs., and its outer diameter 14 inches; find its inner diameter.

40. If the weight of a sphere be 1 cwt., and the diameter 13 inc hes, what is the weight of each of 3 spheres of the same material, whose diameters are 15, 18, and 20 inches?

PROBLEM 5.

Find the volume and surface of a cone or pyramid.



Rule.—Multiply the area of the base of the cone or pyramid by its perpendicular height, and one-third of this product will be the volume.

FORMULA.

Let A = area of the base.

$P\ G = h$ = perpendicular height.

$$\therefore V = \frac{A\,h}{4}.$$

$$= \frac{\pi\ D^2}{4}\ \frac{h}{3}, \text{ where } D \text{ is the diameter of a right cone.}$$

Rule.—Square the diameter and multiply by ·7854; to this product add one-half of the product of the slant side and circumference of the base for the surface.

FORMULA.

$$S = \frac{\pi\ D^2}{4} + \frac{l\,c}{2}, \text{ where } l \text{ is the slant side and } C \text{ is the circumference of the base.}$$

EXAMPLES.

Question 15.—Section 9.

$$\textit{Volume} = \frac{\pi\ D^2\ h}{12} = \frac{3{\cdot}1416 \times 144 \times 8}{12} = 301{\cdot}59 \text{ cubic feet.}$$

$$\text{Surface} = \frac{\pi D^2}{4} + \frac{l\,c}{2} = \frac{3{\cdot}1416 \times 144}{4} + \frac{10 \times 12 \times 3{\cdot}1416}{2}$$

$$= 301{\cdot}59 \text{ square feet.}$$

Let x = the displacement in cubic feet.

$\therefore 62{\cdot}5 \times x = 301{\cdot}59 \times 41{\cdot}06.$

$$\text{or, } x = \frac{301{\cdot}59 \times 41{\cdot}06}{62{\cdot}5} = 198{\cdot}13 \text{ cubic feet.}$$

Section 1.

1. Find the volumes of the three cones, whose dimensions are as follow: diameters, 3, 5, and 8 feet; heights, 15, 8, and 16 feet.

2. Find the volumes of the three cones, whose diameters and heights are as follow: 15 feet, 15 feet 9 inches, and 18 feet 7 inches; and 21 feet, 12 feet 6 inches, and 9 feet 3 inches.

Section 2.

3. Find the volumes of the three cones, whose diameters and heights are 20 feet 5 inches, 14 feet 9 inches, and 17 feet; and 40 feet 6 inches, 33 feet 3 inches, and 18 feet 4 inches.

Section 3.

4. The base of a pyramid is an equilateral triangle, whose side is 2 feet, and height 18 feet; find its volume.

5. The base of a pyramid is a square, whose side is 2 feet, and height 18 feet; find its volume.

6. The base of a pyramid is a triangle, whose three sides are $2\frac{1}{4}$, $3\frac{1}{4}$, and $4\frac{1}{2}$ feet, and height 15 feet 6 inches; find its volume.

Section 4.

7. Find the weight of a cone of Riga fir, whose diameter is 2 feet 3 inches, and height 5 feet 7 inches.

8. Find the weight of a cone of granite, whose diameter is 8 feet 3 inches, and height 4 feet 8 inches.

Section 5.

9. Find the surface and volume of the two cones, whose diameters are 9 feet 3 inches and 8 feet 5 inches, and heights 7 feet 4 inches and 100 feet 4 inches.

Section 6.

10. Find the weight of two cones of **Riga fir**, whose diameters are 12 feet 3 inches and 14 feet, and heights 96 feet 3 inches and 15 feet.

Section 7.

11. Find the displacement and volume of a cone of oak, whose diameter is 18 feet 6 inches, and height 21 feet 10 inches.

Section 8.

12. Find the volume and weight of a cone of lead, whose diameter is 3 feet 4 inches, and height 1 foot 6 inches.

13. Find the volume and displacement of a cone of yellow pine, whose diameter is 36 feet 9 inches, and height 18 feet 9 inches.

Section 9.

14. Find the surface, weight, and volume of a cone of oak, whose height is 3 feet, and diameter 8 feet.

15. Find the displacement, surface, and volume of a cone of teak, whose height is 8 feet, and diameter 12 feet.

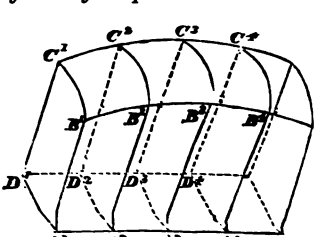
Section 10.

16. Find the volumes of the cones, whose diameters are 4 feet 3 inches, 5 feet 9 inches, and 8 feet 3 inches; and heights 2 feet 8 inches, 18 feet, and 9 feet 4 inches.

Problem 6.

To find the volume of any solid by Simpson's Rule.

Divide the solid into an even number of equal intervals, by the parallel sections $A_2 B_2 C_2 D_2$, &c. Find the area of each section by Simpson's Rule, then apply these areas in the same manner as the ordinates of a plane curve are applied in Simpson's Rule, *the lines* $A_1 A_2 = A_2 A_3$, &c., *being the* common interval. *The result* will be the volume required.



Problem 7.

To find the centre of gravity of a solid by Simpson's Rule.

Use the area of the sections, as described in Problem 6, in the same manner as the ordinates of a plane curve are used in Problem 6, Mensuration of Surfaces.

Problem 8.

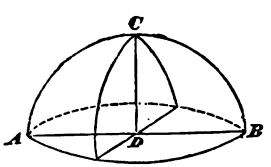
To find the volume of a segment of a sphere and cone, and also the surface of a segment of a sphere.

Segment of Sphere.

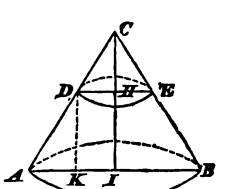
Rule.—From three times the diameter take twice the height of the segment; multiply this difference by one sixth of the product of 3.1416, and the square of the height of the segment, and the result will be the volume.

Segment of Cone.

Rule 1.—Find the line $H\,C$ by similar triangles, as follows :—
$A\,B - D\,E : D\,E :: I\,H : H\,C.$
Find the volumes of the two cones $A\,B\,C$, $D\,C\,E$, and their difference will be the volume of the segment $A\,D\,E\,B$.



Rule 2.—To the squares of the diameters $A\,B$ and $D\,E$ add their product; multiply this sum by .7854 and by one-third of the height $I\,H$, and the result will be the segment.



Surface of Segment of Sphere.

Rule.—Multiply the diameter, height of segment, and 3·1416 together, and the product will be the surface.

Formulæ.

Segment of Sphere.

$$V = \frac{\pi\, h^2}{6}\,(3\,D - 2\,h)$$

Segment of Cone.

$$V = \frac{\pi I H}{12} (A B^2 + D E^2 + A B \cdot D E)$$

Surface of Segment of Sphere.

Surface $= \pi h D$.

EXAMPLES.

Questions 24 and 25.—Section 10.

$$\text{Vol.} = \frac{\pi h}{12} \left\{ D^2 + d^2 + D d \right\} = \frac{3{\cdot}1416 \times 95}{12}$$

$$\left\{ \frac{49}{4} + \frac{25}{4} + \frac{35}{4} \right\} = 677{\cdot}734 \text{ c. ft.}$$

Then the weight $= \frac{677{\cdot}734 \times 47{\cdot}06}{2240} = 14$ tons 4 cwt. 3 qrs.

Again, vol. $= \frac{\pi h^2}{6} (3 D - 2 h) = \frac{3{\cdot}1416 \times 9}{6 \times 4} (16.5 - 3) =$

$15{\cdot}904$ cub. ft.

Then the weight $= 15{\cdot}904 \times 47{\cdot}06 = 748{\cdot}45$ lbs.

Section 1.

1. The diameter of a sphere is 18 feet, and the height of the segment 9 feet; find its volume.
2. The diameter of a sphere is 24 feet, and the height of the segment 14 feet; find its volume.
3. The diameter of a sphere and height of its segment are 23½ feet and 15 feet; find its volume.

Section 2.

4. The diameters of the two ends of a segment of a cone are 10 feet and 6 feet, and the height 15 feet; find its volume.
5. The diameters and height of the segment of a cone are 12 feet, 8 feet, and 24 feet; find its volume.
6. The diameters and height of the segment of a cone are 11 feet, 7 feet, and 36 feet; find its volume.

Section 3.

7. The diameter of a sphere is 7 feet, and the height of the segment 2 feet; find its volume.

8. The diameters of the two ends of a segment of a cone are 7 feet and 3 feet, and its height 18 feet; find its volume.

9. The diameter of a sphere is 8 feet, and the height of the segment 3 feet; find its volume.

10. The diameters of the two ends of a segment of a cone are 11 feet and 8 feet, and its height 15 feet; find its volume.

Section 4.

11. The diameters and height of the segment of a cone are 15, 7, and 15 feet; find its volume.

12. The diameter and height of a segment of a sphere are 13 and 6 feet; find its volume.

13. The diameters and height of the segment of a cone are 5, 2, and 18 feet; find its volume.

14. The diameter and height of the segment of a globe are $4\frac{1}{2}$ and $1\frac{1}{2}$ feet; find its volume.

Section 5.

15. The diameter and height of the segment of a sphere are 28 and 10 feet; find its volume.

16. The diameters and height of the segment of a cone are 32, 23, and 18 feet; required its volume.

17. The diameter and height of the segment of a sphere are 6 and $2\frac{1}{4}$ feet; find its volume.

18. The diameters and height of the segment of a cone are 10, 4, and 36 feet; find its volume.

Section 6.

19. The diameter and height of the segment of a sphere are 8 and 3 feet; find its volume.

20. The diameters and height of a segment of a cone are $3\frac{1}{2}$, $1\frac{1}{4}$, and $9\frac{1}{2}$ feet; find its volume.

Section 7.

21. Find the surface and weight of the segments in Section 1, *supposing each* to be Riga fir.

Section 8.

22. Find the surfaces and weights of the segments of spheres in Sections 3 and 4, supposing each to be English oak.

Section 9.

23. Find the surfaces and weights of the segments of spheres in Sections 5 and 6, supposing each to be lignum vitæ.

Section 10.

24. Find the weight of a mast of Riga fir, whose diameters are 3 feet 6 inches and 2 feet 6 inches, and length 95 feet.

25. Find the weight of the segment of a sphere of Riga fir, whose diameter is 5 feet 6 inches, and height 1 foot 6 inches.

In the segment of a sphere, if $A\,B = b$, and $C\,D = h$ be the given quantities, instead of the diameter and the line $C\,D = h$, the height of the segment, the Rules and Formulæ will be slightly altered.

Rule.—To $\frac{3}{4}$ of the square of the base (b), add the square of the height (h); multiply this sum by the product of the height (h) and 3·1416; then $\frac{1}{6}$ of the result will be the volume of the segment.

Rule.—To $\frac{1}{4}$ of the square of the base (b), add the square of the height (h); multiply this sum by 3·1416, and the product will be the surface of the segment.

FORMULÆ.

$$V = \left(\frac{3\,b^2}{4} + h^2 \right) \times \frac{\pi\,h}{6}$$

$$S = \left(\frac{b^2}{4} + h^2 \right) \pi$$

Section 11.

26. Find the volumes and surfaces of the three segments of spheres, whose bases are 24, 32, and 16 feet, and whose heights are 4, 7, and 5 feet respectively.

Section 12.

27. Find the volumes and surfaces of the three segments of spheres, whose bases are 18, 26, and 28 feet, and whose heights are 5, 9, and 12 feet respectively.

Section 13.

28. Find the volumes and surfaces of the four segments of spheres, whose bases are 12, 15, 21, and 25 feet, and whose heights are 3, 6, 8, and 11 feet respectively.

Section 14.

29. Find the volumes and surfaces of the three segments of spheres, whose bases are 34, 52, and 14 feet, and whose heights are 12, 7, and 10 feet respectively.

Section 15.

30. Find the volumes and surfaces of the three segments of spheres, whose bases are 30, 22, and 28 feet, and whose heights are 15, 13, and 9 feet respectively.

PROBLEM 9.

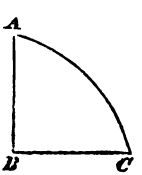
To find the volumes of an ellipsoid and paraboloid.

Ellipsoid.

Rule.—Multiply the three diameters together, and this product again by one-sixth of 3·1416, for the volume required.

Paraboloid.

Rule.—Multiply the square of the radius (*B C*) by the axis *A B*, and this product again by one-half of 3·1416, for the volume required.



FORMULÆ.

Ellipsoid.

Let a, b, c be the diameters.

$$\text{Then, } V = \frac{\pi a b c}{6} .$$

Paraboloid.

$$\text{Then, } V = \frac{\pi A B . B C^2}{2} .$$

If $b = c$, the ellipsoid becomes the prolate spheroid.

If $a = c$, the ellipsoid becomes the oblate spheroid, which is the figure of the earth.

Find the volume of an ellipsoid, whose diameters are 7, 4, a 3 feet.

$$V = \frac{\pi a b c}{6} = \frac{3{\cdot}1416 \times 7 \times 4 \times 3}{6} = 43{\cdot}9824 \text{ cubic feet.}$$

EXAMPLES.

Section 1.

1. Find the volume of an ellipsoid, whose diameters are 25, 1 and 12 feet.
2. Find the volume of a paraboloid, whose axis is 5 feet, a radius of base 3 feet.
3. Find the volume of a prolate spheroid, whose transverse a (a) is 15 feet, and conjugate (b) is 8 feet.
4. Find the volume of an oblong spheroid, whose transverse a (a) is 8 feet, and conjugate 5 feet.

Section 2.

5. Find the volume of an ellipsoid, whose diameters are 25, 1 and 10 feet.
6. Find the volume of a paraboloid, whose axis is 15 feet, a radius of base 7 feet.
7. Find the volume of a prolate spheroid, whose transverse a is 15 feet, and conjugate 9 feet.
8. Find the cubic miles in the earth, whose transverse diamet is 7925½ miles, and conjugate 7899 miles.

Section 3.

9. Find the volume of an ellipsoid, whose diameters are 13½, 8 and 6¾ feet.
10. Find the volume of a paraboloid, whose axis is 31½ feet, a radius of base 15¾ feet.
11. Find the volume of a prolate spheroid, whose transverse di meter is 15 feet 9 inches, and conjugate 9 feet 4 inches.
12. Find the volume of an oblong spheroid, whose transverse di meter is 18 feet 3 inches, and conjugate 12 feet 10 inches.

Section 4.

13. Find the volume of an ellipsoid, whose diameters are 19 feet inches, 16 feet 10 inches, and 12 feet 2 inches.
14. Find the volume of a paraboloid, whose axis is 13 feet 9 inche and radius of base 8 feet 11 inches.

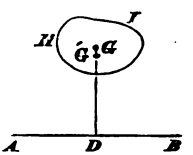
15. Find the volume of a prolate spheroid, whose transverse diameter is 7 feet 10 inches, and conjugate diameter is 3 feet 10 inches.

16. Find the volume of an oblong spheroid, whose transverse diameter is 8 feet 9 inches, and conjugate diameter is 4 feet 7 inches.

Problem 10.

Guldinus's method of finding the volumes and surfaces of solids of revolution.

Let G be the centre of gravity of the curved area $H\ I$, and draw $G\ D$ perpendicular to $A\ B$.



If the area $H\ I$ revolve round $A\ B$ as an axis, a solid of revolution will be generated.

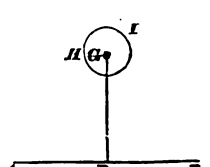
Guldinus has proved that the volume of this solid of revolution is equal to a prism, whose base is the revolving figure $H\ I$, and height the length described by the centre of gravity G.

Let G′ be the centre of gravity of the curved line $H\ I$.

Then the surface of the solid of revolution is a rectangle, one side of which is the length of the generating curve $H\ I$, and the other the length described by the centre of gravity G'.

This method is very useful in finding the volumes and surfaces of revolution, when the centre of gravity is readily found; and conversely, when the volumes and surfaces of revolution are known, the centres of gravity of the generating areas can readily be obtained.

If $H\ I$ be a circle, G is its centre, and the solid generated will be a cylindrical ring.



Let $d = 4$ feet, the diameter of the generating circle $H\ I$. Put $D\ G = 8$ ft.

Area of circle $= \cdot 7854 \times 16 = 12 \cdot 5664$.

Length described by centre of gravity $G = 16 \times 3 \cdot 1416$.

$= 50 \cdot 2656$.

Volume of ring $= 12 \cdot 5664 \times 50 \cdot 2656 = 631 \cdot 659$ cubic feet.

To find the surface of the ring, the centre of gravity of the area *and the circumference of* $H\ I$ is evidently in the centre.

The length of the circumference $H I = 3 \cdot 1416 \times 4 = 12 \cdot 5664$ feet.

Length described by the centre of gravity $= 16 \times 3 \cdot 1416 = 50 \cdot 2656$ feet.

Surface of volume $= 12 \cdot 5664 \times 50 \cdot 2656 = 631 \cdot 659$ square feet.

In this case the surface of the ring is equal to its volume.

Examples.

Section 1.

1. Find the volume and surface of a ring, whose thickness is 3 inches, and inner diameter 16 inches.

2. Find the volume and surface of a ring, whose thickness is 6 inches, and inner diameter 9 inches.

Section 2.

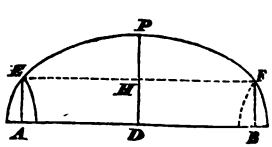
3. Find the weight of a ring, of wrought iron, whose thickness is $1\frac{3}{4}$ inches, and inner diameter $6\frac{3}{4}$ inches.

4. Find the volume and surface of a ring, whose thickness is 8 inches, and inner diameter 18 inches.

Section 3.

Let the area $A\ E\ P\ F\ B$ revolve about the line $A\ B$, forming the middle frustrum of a circular spindle.

A variety of casks are of this figure.



Let A = area of the segment $E\ P\ F$.

H = diameter at the bung $P\ D$.

h = diameter at the head $A\ E$.

$a = A\ B$.

$$\text{Volume} = \frac{\pi}{4}\left\{\frac{a}{3}\left(2\,a^2 + 3\,h^2\right) + 2\,(H+h)\,A - \frac{2\,a^2\,A}{H-h}\right\} \quad -(B)$$

If $h = 0$, then the figure becomes a circular spindle, whose

$$\text{vol.} = \frac{\pi}{2}\left\{\frac{a^2}{3} + HA - \frac{a^2 A}{H}\right\}.$$

The following is an approximate formula instead of B, and in most cases it will not be far from the truth.

$$\text{Approximate vol.} = \frac{\pi}{12}(2H^2 + h^2)\,a.$$

5. Find the number of gallons in a cask, whose bung and head diameters are 18 feet 10 inches, and length 24 inches.

In this case $H = 18$, $h = 10$, $a = 24$, and $PH = 4$.

$$\text{Hence, } A = \text{area } EPF = \left\{7\sqrt{EH^2 + \frac{2PH^2}{7}} + \frac{4}{3}\sqrt{EH^2 + PH^2}\right\} \times \frac{4PH}{25} = 65{\cdot}392.$$

See Problem 8, Mensuration of Surfaces.

$$\text{Therefore, vol.} = \frac{\pi}{4}\left\{\frac{a}{3}(2a^2 + 3h^2) + 2(H+h)A - \frac{2a^2 A}{H-h}\right\}$$

$$= \frac{3{\cdot}1416}{4}\left\{\frac{24}{3}(2 \times 24^2 + 3 \times 10^2) + 2(18 + 10)\,65{\cdot}392 - \frac{2 \times 24^2 \times 65{\cdot}392}{18 - 10}\right\}$$

$$= \frac{3{\cdot}1416}{4}\left\{11616 - 5754{\cdot}496\right\}.$$

$$= 4603{\cdot}625 \text{ cubic inches.}$$

$$\text{The number of gallons} = \frac{4603{\cdot}625}{277{\cdot}274} = 16{\cdot}603.$$

6. Find the number of gallons in a cask, whose bung and head *diameters* are 20 and 14 inches, and length 38 inches.

Section 4.

7. Find the volume of a circular spindle, whose height is 5 feet, and base 26 feet.

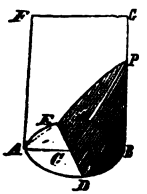
8. Find the number of gallons in a cask, whose bung and head diameters are 24 and 18 inches, and length 48 inches.

Miscellaneous Formulæ.

Cylindrical Ungula.

Vol. of $CDBPE =$

$$\left\{\frac{\frac{2\,CD^3}{3} - A\,(r - BC)}{BC}\right\} BP.$$



Where r = radius of the cylinder.

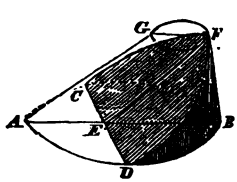
And A = area of $EBDC$.

Surface of $DBPE = \left\{\frac{2\,r.CD - a\,(r - BC)}{BC}\right\} BP.$

Where a = arc EBD.

Conical Ungula.

Let $ACBD$ be the base of any cone or pyramid; put A = area of $CBDE$ and B area of $CFDE$.



Vol. of $CFBDE = \frac{FH}{3\,(AB - GF)}$

$$\left\{A.AB - \frac{GF.BE.B}{EF}\right\}.$$

In a right cone $CBDE$ is the segment of a circle, whose diameter is AB; and $CFDE$ is an ellipse, parabola, or hyperbola, according as FEB is less, equal to, or greater than GAB.

Surface of $CFBDE = \frac{BF}{BH.EF}\left\{A.EF - B.EH\right\}.$

Prolate Spheroid.—Put $e^2 = \frac{a^2 - b^2}{a^2}$, where a and b are the semi-axis of the generating ellipse.

$$\text{Surface} = 2\pi a^2 \left\{ \sqrt{1 - e^2}\, \frac{\text{Sin.}^{-1} e}{e} + 1 - e^2 \right\}.$$

Oblate Spheroid.—

$$\text{Surface} = 2\pi a^2 \left\{ 1 + \frac{1 - e^2}{2e} \log. \left(\frac{1 + e}{1 - e} \right) \right\}.$$

Paraboloid.—Put m = focal distance, and x = abscissa.

$$\text{Surface} = \frac{8\pi \sqrt{m}}{3} \left\{ (x + m)^{\frac{3}{2}} - m^{\frac{3}{2}} \right\}.$$

Spherical Triangle.—Let A, B, C be the three angles, and S equal one quarter of the surface of the sphere.

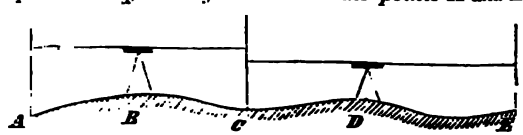
$$\text{Area of triangle} = \frac{A + B + C - 180}{180} . S.$$

The quantity $A + B + C - 180$ is called the spherical excess.

PRINCIPLES OF LEVELLING.

Levelling is generally performed by a spirit-level, a graduated staff, and Gunter's chain, which is used to measure the distances between the several stations.

Required the difference of level between the points A and E.



The level is set up and adjusted at *B*, and the staff is held upright at *A*, the reading of which, in feet and decimals of a foot, is entered in the level-book, in the column marked *back sights*. While the level is turned round and adjusted, the lega remaining still fixed, the staff is carried to *C*, the reading of which is entered in the level-book, in the column marked fore sights. The distance *A C*, measured in chains, is entered in the level book, in the column marked *distances* in the same line, then the level is removed to the station *D*, while the staff at *C* remains fixed.

The operation is then repeated between *C* and *E*, and also to any extent required, observing that, when practicable, the level should be nearly half the distance between the two stations of the level staff. It is not necessary to place the level in a line between the two stations, any convenient situation to set the level will give equally correct results.

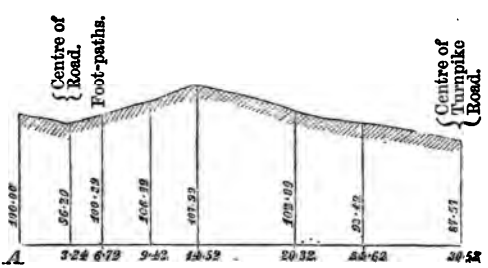
Sketch of a Practical Level Book.

(Datum 100 feet below the bench mark at *A*.)

Back Sights.	Fore Sights.	Rise.	Fall.	Levels reduced to datum line.	Distances.	Remarks.
Ft.	Ft.	Ft.	Ft.	Ft.	Chains.	
				100.00		[(occupation).
4.62	5.42		3.80	96.20	5.24	Centre of road
7.48	8.24	4.18		100.38	6.72	Foot-path.
12.62	3.72	6.60		107.00	9.42	
9.72	8.62	1.10		105.90	14.72	
4.72	10.62		3.90	109.80	20.82	
3.72	14.62		8.00	101.80	24.62	[pike road.
6.62	12.21		3.92	97.88	30.32	Centre of turn-
50.45	62.88	11.78	24.22			
			12.43	total fall.		

When the back sight, 4·62 for instance, is less than the fore sight 8.42, the difference 3.80 denotes the fall between the two stations, and must be entered in the column marked fall; but when the back sight is greater than the fore sight, then the difference denotes the rise between the two stations, and must be placed in the column marked rise. The columns marked, back sights, fore sights, rise, and fall, must be added up, and the difference between the total fall and total rise will be the total rise or fall between the two stations which has been levelled. If the arithmetical operations are correct, the difference between the columns marked back sights and fore sights will be exactly the same as the difference between the columns marked rise and fall. The levels reduced to the datum line, which is, in this case, a line 100 feet below the centre of occupation-road, are performed by adding or subtracting accordingly as the dimension is in the column rise or fall. When the work is correct, the difference between the datum and the level at the last station is equal to the whole rise or fall as the case may be.

In plotting from the reduced levels, as shown in the level book, it is usual to place the distances in a horizontal, and the heights in a vertical line, observing that the scale of distances is much less than the scale of heights, in order to show the undulations of the ground more distinctly.



The above section is sufficiently obvious from the sketch. Of course there are many practical points both in surveying and levelling, which can only be learnt in the field from an experienced surveyor.

LAND SURVEYING.

Gunter's chain is used for measuring the distances which are required in a survey: it is 22 yards long, and consists of 100 links joined together by rings for the purpose of adjusting the chain as it becomes extended by much use.

Each tenth link carries a piece of brass with notches corresponding to the number of tens, and the fiftieth link is marked with a round piece of brass; by this means any distance along the chain is easily ascertained.

With Gunter's Chain there are ten arrows each about 12 inches long, and pointed at one end, so as to be readily pressed into the ground. The line, which is to be measured, is generally ranged by means of convenient rods painted alternately black and white, and sometimes a red flag at the top; so that the rod may be seen at a considerable distance.

The *chain leader* takes the proper end of the chain with the ten arrows, and he is directed in the line to be measured by the *chain director*; and at the end of each chain, the chain leader sticks down an arrow, which is gathered up in regular succession by the chain director, who can readily ascertain the distance measured by the number of arrows which he has in his hand.

When 10 chains has been measured, the whole of the arrows are returned to the chain leader, and the same operation is repeated until the chain leader arrives at the end of the line which is to be measured. Each page of the field-book is usually divided into three colums, the middle one is to enter the distances measured on the chain, where fences or roads are crossed, or offsets, stations, or other marks are made, and the right or left columns are to enter the offsets, and any observations, accordingly as they are on the right or left of the line which is measured by the chain.

Offsets are measured by an offset staff, 10 links long, or a tape-line when nothing better can be obtained.

The perpendicular lines to the chain-line are set off by means of the cross.

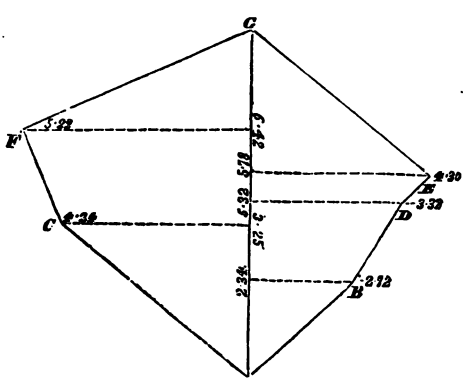
Examples.—Section 1.

(1.) What is the plan and area of a field from the following notes?

Perpendiculars on left.	Base or Station line.	Perpendiculars or right.
	to ⊙ *G*	
	8.32	
F 5.22	6.42	
	5.78	*E* 4.30
	5.32	*D* 3.32
C 4.24	3.25	
	2.34	*B* 2.72
Begin	at ⊙ *A*	and range North.

The symbol ⊙ means at station *A*. And the bearing of the first line in a survey is generally taken; in this case it is North.

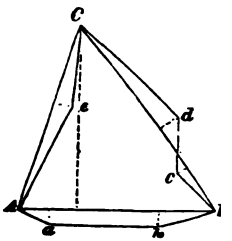
In plotting surveys the uper part of the paper on which the plot is made should be North, if possible.



The plan is obvious, and the area of the field will be the area of the four triangles and the three trapeziums, which is 4 a. 2 r. 19.8 p.

(2.) Plot, and find the area of the fields from the follo notes.

	⊙ *A*	
	8.717	
e .83	3.42	
From	⊙ *C*	
	⊙ *C*	
	10.234	
	4.62	*d* 1.24
	3.05	Fence
c .56	2.34	
From	⊙ *B*	
	⊙ *B*	
	8.32	
	5.60	*b* .34
C 8.37	2.43	
	1.24	*a* .78
From	⊙ *A*	along *A B*.



The plotting is obvious the figure, and the area field *A a b B c d C e* i 3 r. 11p.

Section 2.

(3.)

	⊙ *D*	
	19.91	
.84	15.32	
	13.24	Fence
	7.84	1.72
	4.57	Fence
.52	3.58	
	0.00	range to *D*
From	⊙ *C*	
	⊙ *B*	
	46.24	
	40.32	2.34
6.32	36.24	
	30.42	4.32
D 11.52	27.34	
	15.62	8.32
	12.34	5.72
C 5.32	8.42	4.62
3.24	5.67	2.34
	0.00	
From	⊙ *A*	range to B

(4.)

	⊙ *D*	
	22.10	
1.24	15.27	
3.44	9.32	
2.34	5.63	
1.24	3.45	
	0.00	
From	⊙ *B*	range
	⊙ *D*	
	24.17	
	17.24	7.34
	13.42	10.52
B 15.32	8.24	
	4.73	2.32
	0.00	
From	⊙ *A*	range t

Section 3.

(5.)			(6.)		
	⊙ *A*				
	16.37				
	12.31	2.14			
	8.32	1.52		⊙ *A*	
	0.00			12.80	
From	⊙ *C*	range to *A*		6.22	1.24
	⊙ *C*			0.00	
	24.16		From	⊙ *C*	range to *A*
	18.23	1.16		⊙ *C*	
	12.42	3.14		14.95	
	5.72	2.23		8.42	3.34
	0.00			0.00	
From	⊙ *B*	range to *C*	From	⊙ *B*	range to C
	⊙ *B*			⊙ *B*	
	24.32			15.42	
	15.34	2.31		7.86	2.34
	12.34	Fence	*C* 11.42	5.76	
1.43	7.83			0.00	
C 15.36	5.67	Fence	From	⊙ *A*	range to *B*
	3.46	.84			
	0.00				
From	⊙ *A*	range to *B*			

Section 4.

(7.)			(8.)		
				⊙ *E*	
				16·78	·00
				15·26	·28
	⊙ *A*			14·24	·24
	9·565			13·10	·08
	4·54	2.32		11·23	·16
	0·00			10·27	·25
From	⊙ *C*	range to *A*		8·32	·34
	⊙ *C*			7·23	·64
	12·86			4·23	·50
	8·32	1·72		3·42	·74
	0·00			0·00	·00
From	⊙ *B*	range to *C*	From	⊙ *B*	range to *E*
	⊙ *B*			⊙ *F*	
	14·52			36·25	
	8·32	1·42		26·73	4·32 *E*
C 8·32	4·72		*D* 7·24	20·32	
	0·00		*C* 9·34	16·32	
From	⊙ *A*	range to *B*		10·32	7·84 B
				0·00	
			From	⊙ A	range to F

Section 5.

(9.)			(10.)		
	⊙ *A*				
	14·115			⊙ *A*	
	10·42	1·56		12·765	
	7·84	·32		10·32	
	5·76	1·32	1·42	6·98	
From	⊙ *C*	range to *A*	·78	5·62	
	⊙ *C*		·56	⊙ *D*	range to *A*
	19·371		From		
	17·32	·76		⊙ *D*	
	15·92	Fence		10·327	
·64	14·72		1·55	4·62	
	12·31	Fence	From	⊙ *C*	range to *D*
	8·32	1·72		⊙ *C*	
	5·27	Fence		19·169	
1·62	3·14		1·40	17·98	
From	⊙ *B*	range to *C*		15·00	Fence
	⊙ *B*			7·56	1·48
	18·34			6·24	Fence
	15·26	·42	1·32	4·32	
	14·12	Fence	From	⊙ *B*	range to *C*
·74	12·24			⊙ *B*	
	9·72	Fence		20·32	
	8·63	2·36		14·72	·72
	5·67	1·34	*C* 17·42	12·32	
C 13·42	4·37			5·6	1·42
	0·00		*D* 12·32	3·34	
From	⊙ *A*	range to *B*	From	⊙ *A*	range to *B*

Section 6.

(11.)			(12.)		
	⊙ *A*				
	13·541				
·52	12·14				
1·34	10·24				
·34	4·25			⊙ *B*	
From	⊙ *C*	range to *A*		30·42	
	⊙ *C*		·90	26·32	
	15·692			24·23	1·00
	9·32	1·42	1·32	20·32	1·51
	6·31	Fence		14·62	1·84
1·32	5·62		3·22	12·62	
	4·12	Fence	2·34	10·32	2·24
	3·24	·84	·84	5·76	
From	⊙ *B*	range to *C*	1·24	3·42	
	⊙ *B*			1·24	3·24
	15·34			0·00	
	10·32	·50	From	⊙ *A*	range to *B*
C 12·32	5·62				
	4·72	1·43			
From	⊙ *A*	range to B			

Section 7.

The offsets to be calculated by Simpson's Rule.

(13.)

	⊙ *A*	
	18·00	·00
	15·00	·13
	12·00	·18
	9·00	·25
	6·00	·20
	3·00	·14
	0·00	·00
From	⊙ *C*	range to *A*
	⊙ *C*	
·00	12·00	
·12	10·50	
·22	9·00	
·32	7·50	
·50	6·00	
·44	4·50	
·22	3·00	
·10	1·50	
·00	0·00	
From	⊙ *B*	range to *C*
	⊙ *B*	
	16·00	·00
	14·00	·22
	12·00	·54
	10·00	·62
	8·00	·85
	6·00	·70
	4·00	·55
	2·00	·34
	0·00	·00
From	⊙ *A*	range to *B*

(14.)

	⊙ *A*	
	8·00	·00
	7·20	·13
	6·40	·17
	5·60	·25
	4·80	·36
	4·00	·74
	3·20	·55
	2·40	·30
	1·60	·24
	·80	·10
	·00	·00
From	⊙ *C*	range to *A*
	⊙ *C*	
	11·20	·00
	9·80	·25
	8·40	·34
	7·00	·60
	5·60	·75
	4.20	·63
	2·80	·25
	1·40	·17
	0·00	·00
From	⊙ *B*	range to *C*
	⊙ *B*	
	13·00	·00
	11·70	·09
	10·40	·12
	9·10	·32
	7·80	·42
	6·50	·50
	5·20	·24
	3·90	·13
	2·60	·04
	1·30	·02
	0·00	·00
From	⊙ *A*	range to *B*

SLIDE RULE.

The slide rule affords a mechanical method of performing the arithmetical operations of multiplication, division, and the square roots, by the aid of lines drawn upon a two-feet rule. The lines in question denote the logarithms of the natural numbers, and their practical use will be readily explained by any person who understands them, in a much more effective manner than could possibly be done by a written description.

TIMBER MEASURE.

When timber forms regular figures it is measured by the rules corresponding to those figures: but round or unsquared timber is measured by an empirical rule as follows. Multiply the square of the quarter girth by the length.

The quarter girth is obtained by measuring the circumference at several places, and then finding a mean circumference or girth by adding all the girths together, and dividing by the number of them. The mean girth is divided by 4 for the quarter girth.

Example.—What is the content of a tree, which measures 10 feet 7 inches and 6 feet 5 inches at the two ends, and 9 feet in the middle; the length being 24 feet 8 inches.

$$\text{The mean girth} = \frac{\overset{\text{ft. in.}}{10\ 7} + \overset{\text{ft. in.}}{6\ 5} + \overset{\text{ft.}}{9}}{3} = \frac{\overset{\text{ft.}}{26}}{3}.$$

$$\text{The quarter girth} = \frac{26}{12} = \frac{13}{6} \text{ feet.}$$

$$\textit{Then,}\ \frac{13^2 \times 24\frac{2}{3}}{36} = 115{\cdot}8 \text{ c. feet, the content of the tree.}$$

ARTIFICERS' WORK.

All works are measured by the rules corresponding to the figure, or form of them, whether that figure be a triangle, circle, sphere, or cone, &c.

Brickwork.—The standard thickness of brickwork is a brick and a-half; therefore, to estimate the square feet of brickwork, it is only necessary to find the square feet in the figure of the wall, and divide by 9 or 272¼, according as the work is measured by the square yard or the square pole. If the thickness of the wall be greater than the standard thickness, the above result must be multiplied by the thickness of the wall in half-bricks, and divided by 3.

Carpenters' and joiners', as well as plasterers' work, are measured by the square of 100 feet.

Paviors', painters', and masons' work are measured by the square yard. Walls, columns, blocks of stone, &c., are measured by the cubic foot.

Plumbers' work is measured by the pound, or by the hundredweight of 112 lbs.

QUESTIONS ILLUSTRATIVE OF THE APPLICATION OF ALGEBRA TO MENSURATION.

Section 1.

1. What does the formula $V = \frac{L}{6}\left(B\,h + B'\,h' + \frac{B\,h' + B'\,h}{2}\right)$ represent? Find, first, the value of B' in terms of the remaining quantities; and second, the value of h' in terms of the remaining quantities. What does the formula represent when $h' = 0$?

2. What does the formula $V = \frac{\pi\,h^2}{6}(3\,D - 2\,h)$ represent and find the value of D in terms of the remaining quantities?

3. What does the formula $V = \frac{\pi\,h}{6}\left(\frac{3\,b^2}{4} + h^2\right)$ represent? and find the value of b in terms of the remaining quantities and also the diameter of the sphere in terms of V and h.

4. What does the formula $S = \pi\left(\frac{b^2}{4} + h^2\right)$ represent? and find the value of b in terms of S and h, and also the diameter of the sphere.

5. What does the formula $\frac{\pi}{4}(D^2 - d^2)\,L = V$ represent and find the values of D and d in terms of the remaining quantities.

Section 2.

6. Explain the formula $V = \frac{\pi}{6}(D^3 - d^3)$, and find the value of D in terms of V and d.

7. Explain the formula $S = \pi(D^2 - d^2)$, and find the value of D when s and d are given quantities.

8. What does the formula $V = \frac{\pi D^2 h}{12}$ represent? and find the value of D in terms of V and h.

9. Explain the formula $V = \frac{\pi h}{12}\left\{D^2 + D d + d^2\right\}$, and find the value of d in terms of the remaining quantities.

10. What do the formulæ (1) $C = \pi D$, (2) $A = \frac{\pi D^2}{4}$, (3) $\frac{\pi D d}{4}$, (4) $\frac{2 y x}{3}$, represent?

Section 3.

11. The area of a circle is equal to (a) times the circumference; find the area.

12. The area and circumference of a circle are equal to (a) times the diameter; find the area.

13. The inscribed square of a circle is one-half the circumscribing square.

14. The square of the area of a circle is (a) times the circumference added to (b) times the diameter; find the area.

Section 4.

15. The area and circumference of a circle are equal to π; find the area.

16. The diameters of two circles are D and $(D+h)$, and their corresponding areas A and A'; show that $\frac{A' - A}{h}$ is equal to $\frac{\pi}{4}(h + 2D)$.

17. The difference between the circumference and area of a circle is equal to their quotient; find the area.

18. The sum of the circumference and area of a circle is equal to their product; find the area.

Section 5.

19. The area of a circle is equal to (a); find the diameter.

20. The diameter, circumference, and area of a circle are in geometrical progression; find the area.

21. The area of a circle is equal to its circumference added to (a) find the area.

Section 6.

22. The area of a triangle is (a), and the base is equal to (s) times the perpendicular ; find the perpendicular.

23. When the circumference, diameter, and area of a circle are in geometrical progression, show that the area in square feet is equal to the diameter in lineal feet divided by π.

24. The circumference, area, and diameter of a circle are in arithmetical progression ; find the area.

25. The area of an equilateral triangle is equal to its circumference ; find the area.

Section 7.

26. The area of a triangle is equal to (n) times the perpendicula added to the base, and the base is equal to $\left(\frac{1}{n}\right)$ the per pendicular ; find the area.

27. The base, perpendicular, and area of a triangle are in geo metrical progression ; find the area when the common rati is r.

28. The base, perpendicular, and area of a triangle are in arith metical progression ; find the area when the commo difference is d.

Section 8.

29. The perpendicular, circumference, and area of an equilater triangle are in geometrical progression ; find the side.

30. The area of a concentric circle is equal to the sum of th inner and outer circumferences ; find the diameters whe their product is 21.

31. The sum of the base and perpendicular of a triangle is equ to its area ; find the area when the ratio of the base to th perpendicular is equal to n.

32. The difference between the base and perpendicular of triangle is equal to $\left(\frac{1}{n}\right)$ of its area ; find the perpendicu lar when the area is equal to $\left(\frac{n}{2}\right)$.

Section 9.

33. The area of a concentric circle is equal to (a) ; find the outc diameter when it is equal to (n) times the inner diameter

34. The area of a concentric circle is equal to the difference between the diameters, and the ratio of the greater to the less diameter is π; find the area of the concentric circle.

35. The less diameter, greater diameter, and the area of a concentric circle are in arithmetical progression, whose common difference is δ; find the area.

36. The less diameter, greater diameter, and the area of a concentric circle are in geometrical progression, whose common ratio is (r); find the area.

Section 10.

37. The volume of a concentric cylinder, whose base is a circle, is equal to the sum of the squares of the outer and inner surfaces; find the volume of the cylinder when $d^2 = 1 - 4\pi L$. L being its length.

38. The inner surface, outer surface, and volume of a concentric cylinder, whose base is a circle, are in geometrical progression, whose common ratio is r; find the volume whose length is L.

39. The inner surface, outer surface, and volume of a concentric cylinder, whose base is a circle, are in arithmetical progression, whose common difference is δ; find the volume whose length is L feet.

Section 11.

40. The volume, diameter, and height of a right cone are in geometrical progression; find the volume of the cone when the common ratio is r.

41. The area of the base of a right cone, the height, and volume, are in arithmetical progression, whose common difference is δ; find the volume.

42. The diameter of the base of a right cone, the slant height, and the height are in arithmetical progression; find the volume when the common difference is $-\delta$.

Section 12.

43. The volume of a sphere is equal to the sum of its surface and diameter; find the volume and surface.

44. The diameter, volume, and surface of a sphere are in geometrial progression; find the ratio and volume.

45. Divide the diameter of a sphere into two parts, so that the *shell shall* be equal to $\frac{1}{n}$ part of the sphere.

Section 13.

46. The diameter, volume, and surface of a sphere are
metical progression; find the volume and surface.

47. The volume of a segment of a sphere is equal to its
find the volume in terms of the height of the seg

48. A person being elevated (h) feet above the earth'
show that the extent of surface which is visible is
by $\frac{\pi h D^2}{2h+D}$, where D is the diameter of the earth
he is raised to the height of $\frac{D}{n-2}$ feet, he will
of the earth's surface.

Section 14.

49. The diameter of the base, height, and volume of
of a sphere are in geometrical progression, who
(r); find the volume.

50. The diameter of the base, height, and surface of
of a sphere are in arithmetical progression, whos
difference is δ; find the surface.

51. If a person be elevated (h) yards, and can then see
yards on the surface of the earth, show that th
of the earth in yards is determined from th

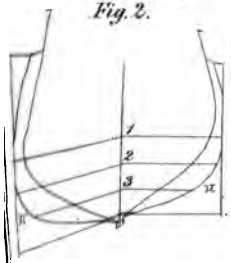
$$\frac{\sqrt{S}}{2\pi h}\left\{\sqrt{S}+\sqrt{S+8\pi h^2}\right\}.$$

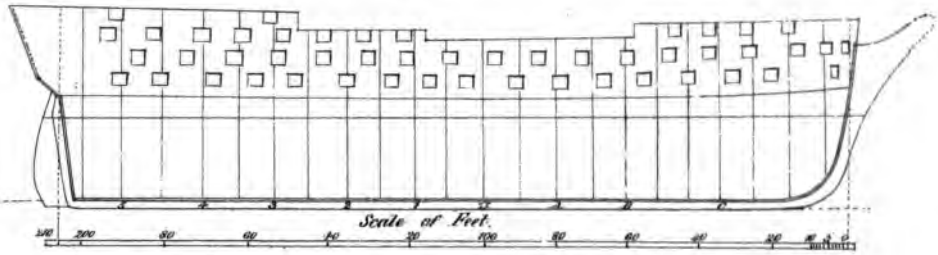
52. If a person be elevated (h) yards, and can ther
distance of (T) yards in every direction, sho
diameter of the earth is $\frac{T^2}{h}-h$.

PLATE I.

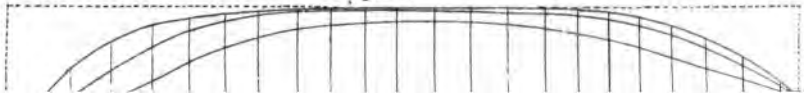
Sheer Plan.
Fig. 1.

Body Plan,
Fig. 2.



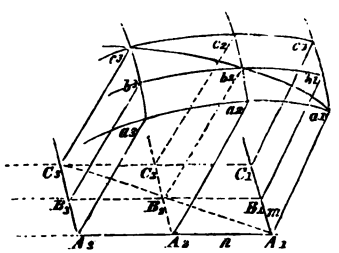


Half Breadth Plan.
Fig. 3.



DR. WOOLLEY'S METHOD OF CALCULATING THE IMMERSED VOLUME OF A SHIP DIRECTLY FROM THE ORDINATES.

A ship is usually divided into equi-distant athwartship, or transverse vertical planes, which are projected on the *Body Plan*, on which the position of any point of the ship is fixed with respect to height and breadth. The planes $A_1\ c_1$, $A_2\ c_2$, $A_3\ c_3$, &c., are portions of these athwartship or transverse vertical sections.



The vessel is also divided into equi-distant horizontal planes, which are projected on the *Half-breadth Plan*, on which the position of any point of the ship is fixed for length and breadth. The planes $A_1\ a_3$, $B_1\ b_3$, $C_1\ c_3$, &c., are portions of these horizontal planes.

The lines $A_1\ a_1$, $A_2\ a_2$, &c., $B_1\ b_1$, $B_2\ b_2$, &c., $C_1\ c_1$, $C_2\ c_2$, &c., are the half-breadth lines, on the horizontal sections $A_1\ a_3$, $B_1\ b_3$, &c., respectively, which extend from the middle of the ship to the outer part a_1, b_1, &c., and are called *ordinates*. There must be an *odd* number of transverse and horizontal planes so as to supply an *odd* number of *ordinates* in each horizontal plane. The position and number of these vertical or transverse and horizontal planes are fixed at the discretion of the constructor, so that $A_1\ A_2 = A_2\ A_3$, &c., $= n$, and $A_1\ B_1 = B_1\ C_1$, &c., $= m$.

The figs. 1, 2, 3 in the accompanying plate (1), will show the *Sheer Plan*, *Body Plan*, and *Half-breadth Plan* of a ship, with their corresponding sections projected upon them. (See Fincham's "Outline of Ship-building," page 32.)

Let the ordinates

$A_1\ a_1 = a_1$ ..... $B_1\ b_1 = b_1$ ..... $C_1\ c_1 = c_1$, &c., ... $L_1\ l_1 = l_1$

$A_2\ a_2 = a_2$ ..... $B_2\ b_2 = b_2$ ..... $C_2\ c_2 = c_2$, &c., ... $L_2\ l_2 = l_2$

$A_3\ a_3 = a_3$ ..... $B_3\ b_3 = b_3$ ..... $C_3\ c_3 = c_3$, &c., ... $L_3\ l_3 = l_3$

&c. &c. &c. &c. &c. &c. &c. &c.

$A_p\ a_p = a_p$ ..... $B_p\ b_p = b_p$ ..... $C_p\ c_p = c_p$, &c., ... $L_p\ l_p = l_p$

where p and the last horizontal section $L_1\ l_3$, are odd numbers

Now, when the points a_1, b_1, &c., of the elementary solid $A_1\,c_3$, are situated on the paraboloidal surface, whose equation is represented by

$$z = A + B\,x + C\,x^2 + D\,y + E\,y^2 + F\,x\,y + G\,x^2\,y + H\,x\,y^2 + I\,x^2\,y^2 \quad (1)$$

where A, B, C, &c., are constants to be determined, so that the surface (1) shall pass through the nine points a_1, b_1, &c., and the x, y, and z are measured on the axis of $A_1\,A_3$, $A_1\,C_1$, $A_1\,a_1$, respectively.

Then, the elementary solid

$$A_1\,c_3 = \frac{m\,n}{9}\left\{a_1 + 4\,a_2 + a_3 + 4\,b_1 + 16\,b_2 + 4\,b_3 + c_1 + 4\,c_2 + c_3\right\}$$

a result which coincides with Simpson's rule as usually adopted by naval architects.

If, however, the points a_1, b_1, &c., of the elementary solid $A_1\,c_3$ be situated on the paraboloidal surface, whose equation is represented by

$$z = A + B\,x + C\,x^2 + D\,y + E\,y^2 + F\,x\,y \ldots\ldots \quad (2)$$

where A, B, and C, &c., are constants to be determined, so that the surface (2) shall pass through the six points a_1, a_2, b_2, a_3, b_3, and c_3,

Then, the elementary solid

$$A_1\,c_3 = \frac{2\,m\,n}{3}\,(a_2 + b_1 + 2\,b_2 + b_3 + c_2)$$

which is an elegant formula, and which was first obtained by the Rev. Dr. Woolley, who derived from it the following rule for the displacement of a ship:—

Take the sum of all the EVEN *ordinates in the first and last horizontal planes; call this result A.*

Take the sum of all the EVEN *ordinates in the 3rd, 5th, 7th, &c., horizontal planes, except the first and last, and multiply this sum by 2, and call this result B.*

Take the sum of the FIRST *and* LAST *ordinates of all the* EVEN *horizontal planes; call this result C.*

Take the sum of all the ordinates, except the first and last, of all the EVEN *horizontal planes, and multiply this sum by 2; call this result D.*

Add the results A, B, C, and D together, and multiply this sum by two-thirds of the product of the common intervals between the vertical and horizontal planes; the result will be the volume required.

The following example will show the application of Dr. Woolley's rule, and the ordinates in Simpson's rule which are omitted.

HORIZONTAL SECTIONS ONE FOOT APART.

	1	2	3	4	5	6	7	8	9	10	11	12	13	sum of first ordinates.
1		3.40		2.50		1.65		1.05		.55		.30		9.50
2	8.85	8.10	7.25	6.45	5.55	4.65	3.80	2.95	2.15	1.50	.95	.55	.40	
3		11.55		9.85		7.50		4.75		2.35		.75		
4	14.05	13.55	12.95	12.15	11.15	9.80	8.25	6.35	4.55	2.90	1.65	.85	.50	
5		14.65		13.50		11.55		7.80		3.40		1.00		
6	15.60	12.20	14.70	14.20	13.45	12.40	11.00	9.05	6.35	3.95	2.20	1.10	.50	
7		15.50		14.50		12.90		9.70		4.35		1.25		
8	15.80	15.45	15.00	14.50	13.85	12.95	11.70	9.95	7.40	4.45	2.40	1.25	.50	
9		15.35		14.35		12.90		9.85		4.35		1.25		
10	15.65	15.25	14.75	14.20	13.50	12.55	11.30	9.50	7.00	4.20	2.50	1.25	.60	
11		15.05		13.90		12.10		8.65		4.00		1.25		
12	14.90	14.55	14.00	13.30	12.30	11.10	9.45	7.40	5.15	3.45	2.25	1.20	.65	
13		13.70		12.05		9.15		5.25		2.25		1.00		
14	13.15	12.40	11.40	10.05	8.50	6.65	4.85	3.45	2.45	1.70	1.20	.80	.60	
15		9.65		6.50		3.65		1.90		1.15		.65		sum of last ordinates.
16	7.10	5.10	3.55	2.60	1.90	1.40	1.10	.95	.75	.70	.60	.55	.50	
17		.85		.60		.50		.50		.50		.45		3.40
A	105.10	195.05	93.60	172.10	80.20	141.25	61.45	97.50	35.80	44.70	13.75	14.70	4.25	
B	105.10	390.10	187.20	344.20	160.40	282.50	122.90	195.00	71.60	89.40	27.50	29.40	4.25	2009.55
														12.90
														2022.45

Sum of *B*.

Sum of first & last ordinates.

The column *A* is the sums of the vertical columns except the first and last ordinates. The column *B* is each number in column *A* multiplied by 2, except the first and last.

Then, $\frac{2}{3} \times 6 \times 2022.45 = 8089.8$ cubic feet, which is the half displacement. Simpson's rule gives 8087.21 cubic feet, which differs from the result of Dr. Woolley's rule by the small quantity of 2.59 cubic feet.

When the ordinates are arranged, as in the above example, Dr. Woolley's method of computing the displacement of a ship will only occupy a few minutes, while it gives a result which differs from that of Simpson's rule only by a small fraction of a ton.

METHOD, BY SIMPSON'S RULE, OF CALCULATING THE DISPLACEMENT OF A SHIP AND ITS CENTRE OF GRAVITY, AS USUALLY ADOPTED BY NAVAL ARCHITECTS.

The sheer plan, Fig. 1, Plate 2, shows the method of dividing the ship into vertical sections, and Fig. 2 gives a correct idea of one of these sections.

In the following scheme, the horizontal lines 1, 2, 3, &c., are the half-breadth lines, or ordinates, in the 1st, 2nd, 3rd, &c., horizontal planes. The small areas forward and aft are taken to estimate the solids (*a*) and (*b*) Fig. 1, which are not comprised between the vertical sections, 1 and 17. The figures 3 and 4 show the general type of the sections from which these areas forward and aft are computed. The small areas below are the areas of the keel (*a*, Fig. 2), below the last horizontal section 13. The small solids, ·25 and 1·3, are outside the vertical planes 1 and 17, and below the horizontal plane 13, and obtained from the half-breadth and sheer plans.

Each horizontal section from 1 to 17 is computed by Simpson's rule; to the result of which is added the small areas forward and aft, and this sum forms the half areas.

The horizontal line, areas below, from 1 to 17 is computed by Simpson's rule; to the result of which is added the solids ·25 and 1·3; the sum 57·55 cubic feet is the solid below the last horizontal section 13.

The vertical half areas, from section 1 to 13, column A, are computed by Simpson's rule; to the result of which the solid 57·55 is added; and the sum 8199·34 cubic feet is the half displacement in cubic feet.

The horizontal half areas, from section 1 to 17, column C, are computed by Simpson's rule; to the result of which 35·22 and 20·91 are added; and the sum is the half displacement in cubic feet. This should agree with the result of the vertical areas nearly, or the operations are not correct.

The column B is the respective half areas multiplied respectively by the consecutive numbers 0, 1, 2, 3, &c.

The column D is the respective half areas multiplied by the consecutive numbers 0, 1, 2, 3, &c.

Multiply the result 32360·9 by the common difference of the horizontal planes, which is 1 in this case, and divide the result by 8199·34 — 57·55 = 8141·79, and the quotient will be 3·974 feet, the distance of the centre of gravity below the first horizontal plane.

Multiply the result 61554·88 by the common difference between the vertical planes, which is 6 feet in this case, and divide the result by 8199·34 — 35·22 — 20·91 = 8143·21, and the quotient will be 45·35 feet, the distance of the centre of gravity abaft the first vertical section.

The position of the centre of gravity has been determined by neglecting the small solids forward, aft, and below

PLATE 2.

Fig. 1.

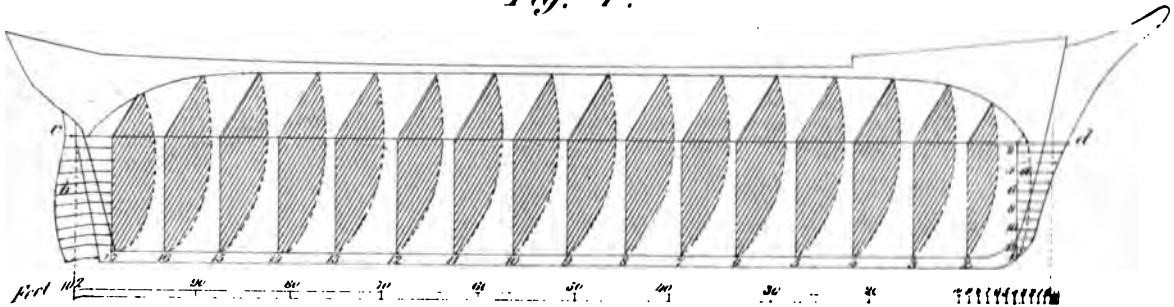
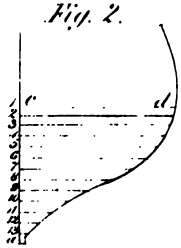


Fig. 2.



Small area aft.
Fig. 4.

Fig. 3.
Small area forward.

Scheme for calculating the displacement and position of the centre of gravity of a ship.

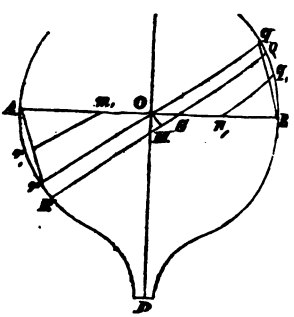
VERTICAL SECTIONS SIX FEET APART.

HORIZONTAL SECTIONS [illegible]

No. of Hor. Sec.	Areas forward.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	Areas aft.	A: Half areas by Simpson's Rule, plus areas for. & aft.	B:	B: Product of half areas by the consecutive numbers 0, 1, 2, &c.
1	8.12	3.95	8.85	12.2	14.05	15.05	15.6	15.8	15.8	15.7	15.65	15.35	14.9	14.2	13.15	11.1	7.1	1.15	1.95	1258.67	0	0
2	6.8	3.4	8.1	11.55	13.55	14.65	15.2	15.5	15.45	15.35	15.25	15.05	14.55	13.7	12.4	9.65	5.1	.85	1.9	1195.80	1	1195.80
3	5.7	2.95	7.25	10.75	12.95	14.1	14.7	15.05	15.0	14.9	14.75	14.55	14.0	13.0	11.4	8.1	3.55	.7	1.9	1125.50	2	2251.00
4	4.6	2.5	6.45	9.85	12.15	13.5	14.2	14.5	14.5	14.35	14.2	13.9	13.3	12.05	10.65	6.5	2.6	.6	1.85	1050.85	3	3152.55
5	3.6	2.05	5.55	8.75	11.15	12.6	13.45	13.8	13.85	13.65	13.5	13.15	12.3	10.7	8.5	4.45	1.9	.55	1.85	960.65	4	3842.60
6	2.7	1.65	4.65	7.5	9.8	11.55	12.4	12.9	12.95	12.9	12.55	12.1	11.1	9.15	6.65	3.65	1.4	.5	1.8	859.80	5	4299.00
7	1.8	1.35	3.8	6.1	8.25	9.95	11.0	11.65	11.7	11.6	11.3	10.55	9.45	7.25	4.85	2.65	1.1	.5	1.7	787.80	6	4426.80
8	1.5	1.05	2.95	4.75	6.35	7.8	9.05	9.7	9.95	9.85	9.5	8.65	7.4	5.25	3.45	1.9	.95	.5	1.6	594.60	7	4162.20
9	1.3	.75	2.15	3.45	4.55	5.3	6.35	7.1	7.4	7.35	7.0	6.15	5.15	3.75	2.45	1.45	.75	.5	1.5	429.90	8	3439.20
10	1.0	.55	1.5	2.35	2.9	3.4	3.95	4.35	4.45	4.35	4.2	4.0	3.45	2.25	1.7	1.15	.7	.5	1.4	274.70	9	2472.30
11	.9	.45	.95	1.4	1.65	1.9	2.2	2.35	2.4	2.45	2.5	2.45	2.25	1.6	1.2	.9	.6	.5	1.3	166.30	10	1663.00
12	.8	.35	.55	.75	.85	1.0	1.1	1.25	1.25	1.25	1.25	1.25	1.2	1.0	.8	.65	.55	.45	1.25	92.65	11	1019.15
13	.6	.35	.4	.5	.5	.5	.5	.5	.5	.55	.6	.6	.65	.65	.6	.55	.5	.45	1.2	52.80	12	633.60
Areas below.	Solid. .25	.2	.25	.3	.35	.4	.45	.5	.5	.55	.6	.65	.7	.8	.85	.9	.95	1.0	Solid. 1.3	Solid feet. 57.55		32360.9 = result of putting col. B into Simpson's rule.
Halfareas by Simpson's Rule plus areas below.	Solid feet. 35.22	19.33	48.73	73.83	91.7	104.01	112.15	116.83	117.56	116.66	114.65	110.46	102.65	87.81	71.1	47.81	23.81	7.9	Solid feet 20.91	8199.34		
		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16				
Product of half the consecutive numbers.		0	48.73	147.66	275.1	416.04	560.75	700.98	822.92	933.28	1031.85	1104.6	1129.15	1053.72	924.3	669.34	357.15	126.4	61554.88	= result of putting col. B into Simpson's rule.		

It is required to find the equal volumes emerged and immersed, when the vessel is inclined through a given angle by the action of the wind and waves : and also the point S.

Let *A D B* be the immersed volume of the vessel before, and *R D Q* the immersed volume after, the vessel has been inclined through the angle *Q S B*. Hence, *A B* is the water-line before, and *R Q* the water-line after, the vessel has been inclined. By the principles of hydrostatics the volumes of the wedges *Q S B* and *R S A* are equal. Draw *r O q* parallel to *R Q*, and bisect *O B* and *O A* in n, and m, and draw $n, q,$ $m, r,$ parallel to *r q*.



By Simpson's rule we have, putting $\angle q\,O\,B = \theta$,

$$\text{Area of } q\,O\,B\,q_1 = \frac{\text{Sin. } \theta}{6}\left\{O\,q + 4\,n_1\,q_1\right\} O\,B \text{ nearly.}$$

$$\text{Area of } r\,O\,A\,r_1 = \frac{\text{Sin. } \theta}{6}\left\{O\,r + 4\,m_1\,r_1\right\} O\,B \text{ nearly.}$$

The lines $O\,q$, $n, q,$ $O\,r$, $m, r,$ and *O B* must be measured for each vertical section on the *Body Plan* ; then the areas indicated above must be obtained for each vertical section on the *Body Plan*, and put into Simpson's rule for the volume of the wedges required.

The following scheme will facilitate the computation of these areas:—

The horizontal lines 1, 3, 6, 9, and 11 are measured by scale from each section on the *Body Plan*.

The horizontal line 2 is obtained by taking four times the horizontal line 1.

The horizontal line 4 is the sum of the horizontal lines 2 and 3.

The horizontal line 5 is the product of the horizontal lines 4 and 6.

The horizontal line 10 is obtained by taking four times the horizontal line 11.

The horizontal line 8 is the sum of the horizontal lines 9 and 10.

The horizontal line 7 is the product of the horizontal lines *6 and 8.*

Vertical Sections being (m) in number, and (n) feet apart.

The values of		1	2	3	4	&c.	$m-3$	$m-2$	$m-1$	m	Values of horizontal lines by Simpson's Rule.
$n_1 q_1$	1										
$4 n_1 q_1$	2										
$O q$	3										$= A$
$O q + 4 n_1 q_1$	4										
$O B(O q + 4 n_1 q_1)$	5										$= q$
$O B$	6										
$O B(O r + 4 r_1 m_1)$	7										$= r$
$O r + 4 r_1 m_1$	8										
$O r$	9										$= B$
$4 r_1 m_1$	10										
$r_1 m_1$	11										

Then, v_1 = volume of the wedge $q\ O\ B\ q_1 = \frac{q \text{ Sin. } \theta}{6}$.

v_2 = volume of the wedge $r\ O\ A\ r_1 = \frac{r \text{ Sin. } \theta}{6}$.

If v_1 is equal to v_2, then $r\ O\ q$ is the water line; but if v_1 is greater than v_2, then the point S is determined from the equation

$$O\ S = \frac{q - r}{6\ (A + B)}.$$

And if v be put equal to the volume of the wedge $Q\ S\ B$, then v is determined from the equation

$$v = \frac{\text{Sin. } \theta}{6} \left\{ \frac{q\ B + r\ A}{A + B} \right\}$$

At angles of 9 or 10 degrees in ships-of-war, of the common form, the line $O\ S$ rarely exceeds 3½ inches. Draw $O\ H$ perpendicular to $R\ Q$, then $O\ H = \frac{v_1 - v_2}{A + B}$ will represent the distance through which the ship has *fallen* in being inclined through the given angle.

To find the longitudinal distance of the centre of gravity of the immersed and emerged volumes from the first vertical section.

It will be necessary to refer to the forgoing scheme for computing the volumes v_1, v_2, and v.

1st.—Multiply the areas of the vertical sections, as given in the horizontal line 5, by the consecutive numbers 0, 1, 2, 3, 4, &c. then put these products into Simpson's rule, and call the result q^1.

Then, $\frac{n\ q^1}{q}$ = the longitudinal distance of the centre of gravity of the volume v_1 from the first vertical section.

2nd.—Multiply the ordinates, as given in the horizontal line 3 by the consecutive numbers 0, 1, 2, 3, 4, &c., then put these *products* into Simpson's rule, and call the result A^1.

Then, $\frac{n A'}{A}$ = the horizontal distance of the centre of gravity of the water-line plane $O\ q$ from the first vertical section.

Then, $H = n \left\{ \frac{(A+B)q' - (q-r)A'}{qB + rA} \right\}$ = the longitudinal distance of the centre of gravity of the immersed volume from the first vertical section.

Perform the same operations upon the horizontal lines 7 and 9.

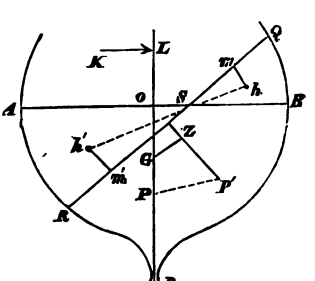
Then, $H' = n \left\{ \frac{(A+B)r' - (q-r)B'}{qB + rA} \right\}$ = the longitudinal distance of the centre of gravity of the emerged volume from the first vertical section.

In the design of a ship the values of H and H' should be as nearly equal as possible, this will be the case when

$$\frac{A+B}{q-r} = \frac{A'-B'}{q'-r'}.$$

STATICAL STABILITY OF A SHIP.

Let $A\ D\ B$ be a vertical section of the ship passing through the centre of gravity P of the part immersed, $A\ B$ being the water-line, and G the centre of gravity of the ship.



It is proved by writers on hydrostatics that the conditions of a floating body are, 1st, that the volume immersed, multiplied by the weight of a cubic foot of water, is equal to the weight of the floating body; 2nd, that the line joining the points P and G is perpendicular to the water-line plane $A\ B$. The first condition was discovered by Archimedes, more than two hundred years before the Christian era, and the second is a consequence easily derived from the properties of parallel forces. *The immersed* volume is usually called the displacement.

Suppose now, a force $K L$ to be applied to L, perpendicular to $G L$, the effect of this force will be to deflect the ship from its upright position $A D B$ to the inclined position $R D Q$, where $R Q$ is now the water-line plane. The mode of determining the position of $R Q$ has been already pointed out.

Let P' be the centre of gravity of the immersed volume after the inclination of the ship, then the upward pressure of the water will be in the direction of $P' Z$ perpendicular to $R Q$; draw $G Z$ at right angles to $P' Z$.

Put $V =$ the volume of water displaced.

$u =$ the density, or the weight of a cubic foot of water.

Then, $u V =$ the weight of the ship $=$ the upward pressure of the water.

And if the ship remains at the inclination to which the force $K L$ has deflected it, then there will result

$$G L \times K L = u V \times G Z, \text{ by the principle of the lever.}$$

The quantity $u V . G Z$ is called the moment of stability, and it is the moment of force by which the ship regains its upright position, after it has been inclined by the wind and waves.

Atwood's theorem for calculating the value of the moment of stability.

Let h and h' be the centres of gravity of the immersed and emerged volumes $Q S B$ and $R S A$ respectively.

Draw $h m$ and $h' m'$ perpendicular to $R Q$, and put $\theta =$ the angle of inclination $Q S B$, and $v =$ the volume of the wedge $Q S B$.

It is not difficult to show that

$u V . G Z = u \left\{ m m' . v - P G . V \text{Sin.} \theta \right\}$ = the moment of stability.

This is Atwood's theorem, which was communicated to the Royal Society and printed in the transactions of 1796 and 1798.

The values of v, V, and the position of P, have already been computed, and there remains the moment $m m' . v$, and the position of the centre of gravity G to be determined in order to find the value of the moment of stability.

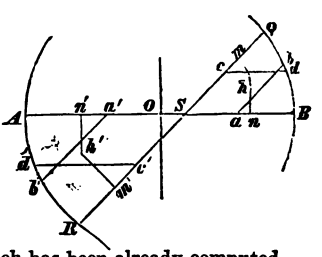
As the exact position of the centre of gravity of a ship is very difficult to ascertain, especially before it is built and fully equipped, it is usual to suppose the centre of gravity to be placed in the water-line plane at *O*. This supposition is sufficient to enable the naval architect to compare, with a tolerable degree of accuracy, the amount of stability in ships of the same class.

Therefore, the moment of stability

$$= u\left\{ m m' . v - P O . V \text{Sin.}\, \theta \right\} \quad (1)$$

The line $h\ h'$ is parallel to $P\ P'$.

To determine the moment $m\ m' . v$.



Bisect the lines $S\ B$, $S\ Q$, $S\ A$, and $S\ R$, in the points a, c, a', and c', and draw $a\ b$, $a'\ b'$ parallel to $R\ Q$, and $c\ d$ and $c'\ d'$ parallel to $A\ B$. Let h and h' be the centre of gravity of the wedges immersed and emerged, draw $h\ n$, $h'\ n'$ perpendicular to $A\ B$, and $h\ m$, $h'\ m'$ perpendicular to $R\ Q$. Put v equal the volume of the equal wedges $S\ Q\ B$ and $S\ R\ A$, which has been already computed.

Then, the moment $(h\ m + h'\ m') . v =$ the values of $(a\ b . S\ B^2 + a'\ b' . S\ A^2)$ for each vertical section, put into Simpson's rule, and the result multiplied by one-sixth of the versed sine of twice the angle θ.

And the moment $(h\ n + h'\ n') . v =$ the values of $(c\ d . S\ Q^2 + c'\ d' . S\ R^2)$, for each vertical section, put into Simpson's rule, and the result multiplied by one-sixth of the versed sine of twice the angle θ.

By means of a table of squares, and a little arrangement of the lines $a\ b$, $S\ B$, &c., for each section, on a similar plan to that adopted in calculating the volume, these moments may be computed in a few minutes.

Now, $m\ m' . v = (h\ n + h'\ n') . v \times \text{Cosec.}\, \theta + (h\ m + h'\ m') . v \times \text{Cot.}\, \theta$.

Therefore the value of the moment of stability is readily obtained from equation (1).

Dynamical Stability.

The dynamical stability of a vessel is the amount of work necessary to be done to heel it through a given angle θ.

Using the same notation and figure as above, Professor Moseley has shown that the

Dynamical stability $= u \left\{ (m\,h + m'\,h')\,v - P\,G.V.\ \text{vers.}\ \theta \right\}$.

And if the dynamical stability of ships of the same class is compared by assuming the centre of gravity to be placed in the point O, then the

Dynamical stability $= u \left\{ (m\,h + m'\,h')\,v - P\,O.V.\ \text{vers.}\ \theta \right\}$.

This formula is readily computed from the data employed to calculate statical stability.

OBSERVATIONS ON THE CONSTRUCTION OF MEN-OF-WAR, BEING AN ABSTRACT FROM FINCHAM'S OUTLINE OF SHIP-BUILDING.

The principal dimensions of a ship are,

1st. *Length on the water-line*, from the fore part of the rabbet of the stem to the after part of the rabbet of the stern-post.

2nd. *Extreme Breadth.*

3rd. *Depth* or draught of water, from the water-line plane to the lower edge of the rabbet of the keel, amidship.

The solid formed by the length, breadth, and depth here *alluded to is* fixed in relation to the weight of the ordnance or *number of guns* in the following manner.

Classes of Ships.	Weight of ordnance in tons.	Ratio of weight of ordnance to the solid formed by the length, breadth, and depth.
ENGLISH NAVY, 1814.		
First rate	487	1 to 519
Second rate	387	1 to 544
Third rate	305	1 to 562
Fourth rate	171	1 to 624
Fifth rate	137	1 to 720
Sixth rate	65	1 to 866
ENGLISH NAVY, 1843.		
First rate	524	1 to 491
Second rate	452	1 to 532
Third rate	350	1 to 565
Fourth rate	247	1 to 692
Fifth rate	161	1 to 761
Sixth rate	85	1 to 941
FRENCH NAVY, 1833.		
First rate	573	1 to 455
90-gun Ship	423	1 to 531
60-gun Frigate	260	1 to 598
46-gun Frigate	146	1 to 659
32-gun Corvette . . .	94	1 to 765
20-gun Brig	50	1 to 840

The determination of the ratio of breadth to length.

There appears some difficulty in fixing the ratio of the breadth to the length of a ship, which must remain, for the present at least, a matter of uncertainty, varying between certain limits.

Three-deckers—The ratio of the breadth to the length varies between 1 : 3·42 and 1 : 3·83.

Two-deckers.—The ratio of the breadth to the length varies between 1 : 3·35 and 1 : 3·98.

Frigates.—The ratio of the breadth to the length varies between 1 : 3·27 and 1 : 3·98.

It is probable that amongst three-decked ships none have been found possessing more generally, and to a greater extent, those qualities which constitute a ship's excellence than the *Caledonia* and the *Victory*. The *Caledonia's* length is 3·83 times its breadth; and the *Victory's* length is 3·62 times its breadth.

The determination of the ratio of the breadth to the depth.

The ratio of the breadth to the depth, or draught of water, varies between the ratio of 1 : ·4 and 1 : ·46.

If the ratio of breadth to length be 1 : 3·65, and the ratio of breadth to depth be 1 : ·41, well formed and good ships may be obtained. In brigs the ratio of breadth to length may be 1 : 3·23 or 3·44; and the ratio of the breadth to depth may be 1 . ·442.

On the midship section.

The form of the midship section will, in a great degree, determine the general form of the body; whilst, upon the particular form of that part of it, which is between the greatest angles of inclination, will depend in a great measure the easy motion of the ship. This section must vary according to the extent or capacity required in the body, and the description of vessel it is intended to construct. The form of a first-rate will be different from that of a cutter; and the midship section of a ship of commerce will be essentially different from that of a yacht, intended only for sailing.

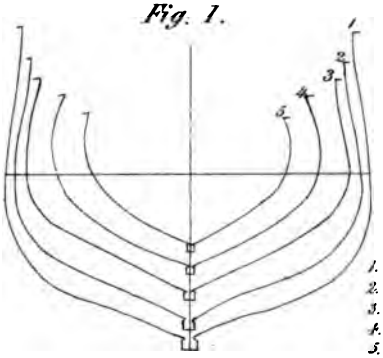
The midship sections, in Fig. 1 (Plate 3), have been obtained by Mr. Fincham from long experience, and by comparing the sections of a large number of ships in connexion with their reported qualities.

Balance sections.

These two sections are parallel to the midship sections, one of which is at one-sixth of the length of the water-line from the *rabbet* of the stem, and the other at the same distance from the *rabbet* of the stern-post. The area of the fore section is to the

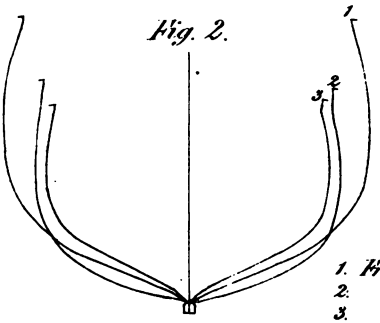
PLATE 3.

Fig. 1.



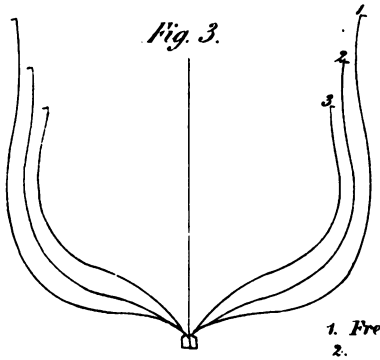
1. 120 Gun Ship.
2. 80 „ „
3. 50 „ Frigate.
4. 20 „ Corvette.
5. 12 „ Brig.

Fig. 2.

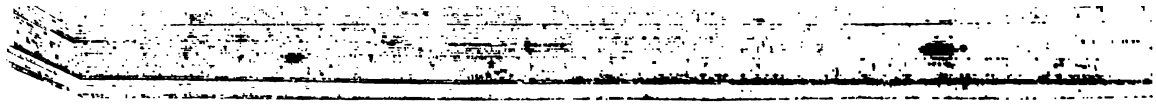


1. French Ship of 100 Guns.
2. „ „ 60 „
3. „ „ 22 „

Fig. 3.



1. French Ship of 100 Guns.
2. „ „ 60 „
3. „ „ 22 „



area of the midship section, for ships of 120 guns, as ·812 to 1; for ships of 90 guns, as ·789 to 1; for 50-gun frigates, as ·77 to 1; for corvettes, as ·723 to 1; and for brigs, as ·682 to 1. And the area of the after section is to the area of the midship section in the same ships respectively as ·731 to 1, as ·719 to 1, as ·698 to 1, ·619 to 1, and as ·526 to 1.

The centre of gravity of the load-water section, in well formed bodies, is about one seventy-fifth of the length of the water-line before the middle.

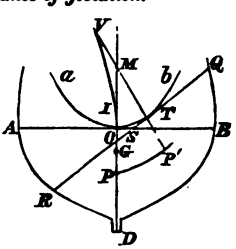
The position of the centre of gravity of displacement should be a definite proportion of the length of the water-line before the middle of that line; in fast sailing ships this proportion varies between ·01 to 1 and ·015 to 1.

For further particulars, and for the formation of the shear and half-breadth plans, see Fincham's "Outline of Ship-building."

The curve described by the centre of buoyancy, the metacentric curve, and the surface of the planes of flotation.

The line $A\ B$ is the water-line when the vessel is at rest, and $R\ Q$ is the water-line when the vessel is inclined to the angle $QSB=\theta$.

During the successive inclinations of the vessel from its upright position to the angle θ, the centre of gravity of immersion, or buoyancy, will describe the curve $P\ P'$, which is called the *metacentric involute.*



Draw $P'\ V$ perpendicular to the curve $P\ P'$ at P', and equal to the radius of curvature. The curve $V\ I$, traced by the successive centres of curvature as the vessel inclines from its upright position to the angle θ, is called the *metacentric curve* by M. Bouguer, but *metacentric evolute* would be more appropriate. Hence, the curve $V\ I$ is the *evolute* of which $P\ P'$ is the *involute.* It would not be difficult to show that $P'\ V$ is perpendicular to the corresponding water-line $R\ Q$.

For the same vessel and displacement the curves $P\ P'$ and $I\ V$ are independent of the centre of gravity G of the vessel, but it is evident that the position of the centre of gravity will affect its stability, which depends upon the perpendicular from G to the line $P'\ V$.

The surface, of which $a\ O\ b$ is a section, that touches the successive water-line planes as the vessel inclines from its upright position to the angle θ, is denominated the surface of the planes of flotation, or the envelope of the water-line planes. When the vessel partakes of a rolling and pitching motion, this surface

ANSWERS TO EXAMPLES IN PROB. 3.

Sec.

1 (1) 5 ft. (2) $bc = 12$, $cd = 6$, and $ad = 4{\cdot}8$.

2 (3) 28 yds. 1 ft. 10 ins. (4) 3 ft. 6·0348 ins.
(5) 153 ft. 11½ ins.

3 (6) 10·995 ins. (7) 4·712 ins.
(8) 2 ft. 2 ins. (9) 1 ft. 6 ins.

4 (10) 100 ft. 9⅜ ins. (11) 7 ft. 8 ins.
(12) 37 ft. (13) 49 ft. 4 ins.

5 (14) 7469·6 and 8403·3. (15) 4 ft. 2 ins.
(16) 5 ft. 4 ins.

6 (17) 24⅝ ft. (18) 966·27 ft.
(19) 342⅜. (20) 3·312, 4·1897 miles.

ANSWERS TO EXAMPLES IN PROB. 4.

Sec.

1 (1) 20 ft. (2) 21·056 ft. (3) 17 ft. 8 ins

2 (4) 48 ft. 9 ins. (5) 255 ft. (6) 132 ft. 9·1

3 (7) 328·45 ft. (8) 118 ft. 6·72 ins. (9) 91 ft. 8·8 i

4 (10) 7 ft. 10·8 ins. (11) 10 ft. 7·684 ins. (12) 12 ft. 2·3 i

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ANSWERS TO EXAMPLES IN PROB. 5.

Sec.

1 (1) 5 ft. 11 ins., 5 ft. 8½ ins., 5 ft. 4 ins.

2 (2) 24 ft. 11⅝ ins.

3 (3) 19 ft. 11·6 ins.

ANSWERS TO EXAMPLES IN PROB. 6.

Sec.

1 (1) ·011 ch., ·044, ·098, ·174, ·271.

2 (2) 11·71875 ft.

(3) From the vertex to focus = ·147, ·04251 ft., ·17007
·3824 ft.

Sec.

3 (4) 3·4327. (5) 5·544 yards.

4 (6) 10·73 ft., 171·69 ft., 268·27 ft.

5 (7) 12·27 miles, 17·36 miles.

6 (8) From the vertex to focus = 180·5, ·0678 ft. ·1994 ft., ·2714 ft.

ANSWERS TO EXAMPLES IN PROB. 7.

Sec.

1 (1) Rad. 52 ft., 41·00 ft. (2) Rad. 33·125 ft., 36·833 ft.

2 (3) Rad. 255 ft., 249·8 ft. (4) Rad. 71·88 ft., 82·44 ft.
(5) Rad. 312 ft., 246·3 ft.

3 (6) Rad. 48·75 ft., 114·6 ft. (7) Rad. 159·964 ft., 116·55 ft.

4 (8) Rad. 260·2 ft. 155·29 ft. (9) Rad. 261·36 ft., 198·75 ft.

ANSWERS TO EXAMPLES IN PROB. 8.

Sec.

1 (1) 32·986 ft. (2) 40·612 ft.

2 (3) 23·039 ft. (4) 9 084 ft.

ANSWERS TO EXAMPLES IN PROB. 9.

Sec.

1 (1) ·97 negative slip. (2) 31·47.
(3) 25·01. (4) 8.6.

2 (5) 11·19. (6) 26·99. (7) 34·58. (8) 24.00.

3 (9) 27·13. (10) 17·75. (11) 16·4. (12) 7·02.

4 (13) 10·77. (14) 23·56. (15) 21·72. (16) 21·82.
(17) 11·3.

MENSURATION OF SUPERFICES OR SURFACES.

ANSWERS TO EXAMPLES IN PROB. 1.

Sec.

1 (1) 630 sq. yds., 5770 sq. ft. (2) 1·81 sq. yds.
(3) 1 a. 2 r. 24 p. 12 yds. (4) 623·62 sq. yds.

2 (5) 2 r. 9 p. 3 yds. (6) 13 a. 1 r. 22 p.
(7) 161 a. 0 r. 15 p.

3 (8) 69 a. 0 r. 23½ p. (9) 93*l.* 12*s.* 3¾*d.*
(10) 5006*l.* 0*s.* 6½*d.*

4 (11) 1672*l.* 9*s.* 9½*d.* (12) 4090⅔ sq. yds.
(13) 57·266 sq. yds.

5 (14) 28·7 sq. yds. (15) 9·5397 sq. ft.

6 (16) 6 ft., area = 9 sq. ft. (17) Base = 2½ ft. area 16½ s

7 (18) Base $\frac{1}{16}$ ft. (19) Area = $4\sqrt{2}$.

8 (20) Perp. = 4⅔, area = 11⅔. (21) 15.
(22) Perp. $(\sqrt{21} - 3)$.

9 (23) 55·99 sq. ft. (24) 178·29 sq. ft.
(25) 8929 ft. 59·04 ins.

10 (26) 53·511 sq. ft. (27) 134·831 sq. ft.
(28) 20·3331.

11 (29) 47·8115 sq. ft. (30) 25·98 sq. ft.
(31) 11·654 ft.

12 (32) 56 ft. 19·8 ins. (33) 97 ft. 45·216 ins.

13 (34) 13 perches 21·566 yds.

14 (35) 70 a. 1 r. 3·104 p.

15 (36) 189½ sq. yds. (37) 1664·5 sq. ft.
(38) 14 ft. 4·6 ins.

16 (39) 1 r. 39 p. 6 yds.

17 (40) 2 r. 33¾ p. (41) 1 a. 1 r. 28½ p.
(42) 302·86 sq. ft.

18 (43) 262*l.* 15*s.* 11*d.*
(44) 343·57 sq. yds.; 446*l.* 12*s.* 9½*d.*

ANSWERS TO EXAMPLES IN PROB. 3.

Sec.

1 (1) 49·8355 ft. (2) 2 r. 28·33 p.
(3) 3*l.* 4*s.* 11*d.*

2 (4) 89·7649 yds.; 26*l.* 3*s.* 7½*d.* (5) 77*l.* 15*s.* 7*d.*

3 (6) 441*l.* 12*s.* 4¼*d.* (7) 73.
(8) 76·125 sq. ft.

4 (9) 2 a. 3 r. 1 p. 6 yds. (10) 73*l.* 8*s.* 9¾*d.*
(11) 4*l.* 6*s.* 6*d.*

5 (12) 76·125, 45·5, 49·835, 52·129.

6 (13) 2 a. 2 r. 15 p. 22⅓ yds., 2 r. 32 p. 16¼ yds., 2 a. 0 r. 1 p. 4¾ yds.

7 (14) 6 a. 1 r. 3·4 p. (15) 4*l.* 10*s.* 1½*d.*
(16) 551*l.* 12*s.* 9*d.*

8 (17) 1·489 ch. (18) 22½ ft.
(19) 40·166 yds.

ANSWERS TO EXAMPLES IN PROB. 4.

Sec.

1 (1) 176.71. (2) 51·8486.
(3) 4995·19. (4) 49·48, 194·82.

2 (5) 70·822. (6) 6 ft. 8 ins., 34·91 sq. ft.
(7) 42·2367, 23 ft. 0⅜ ins.

3 (8) 50·265 sq. ft. 63·617 sq. ft. 176·715 sq. ft.
25 ft. 1½ in. 28 ft. 3¼ in. 47 ft. 1½ in.
33·183 sq. ft. 465·043 sq. ft.
20 ft. 4⅞ ins. 76 ft. 5¼ ins.

4 (9) 113·097 sq. ft. 137·886 sq. ft. 220·353 sq. ft.
37 ft. 8⅜ ins. 41 ft. 7⅛ ins. 52 ft. 7⅜ ins.
351·88 sq. ft. 952·972 sq. ft.
66 ft. 5⅞ ins. 109 ft. 5⅛ ins.

5 (10) 572·556 sq. ft. 1809. 56 sq. ft. 962·115 sq. ft.
84 ft. 9⅞ ins. 150 ft. 9½ ins. 109 ft. 11⅜ ins.
247·45 sq. ft. 565·508 sq. ft.
55 ft. 9⅛ ins. 84 ft. 3½ ins.

6 (11) 490·875 sq. ft. 1590·43 sq. ft. 1885·74 sq. ft.
78 ft. 6⅜ ins. 141 ft. 4⅜ ins. 153 ft. 11¼ ins.
590·36 sq. ft. 1109·38 sq. ft.
86 ft. 1½ in. 118 ft. 0¾ in.

Sec.

7 (12) 11·545 tons. (13) 2 ft. 8·7 ins.
(14) 15 ft. 10¾ ins., 5 ft. 0¾ in.

8 (15) 50·265 sq. ft., 183·78 sq. ft., 285·39 sq. ft., 574·75 sq.

9 (16) 6 a. 1 r. 17·878 p. (17) 5·941 yds.
(18) Diam. = 55·51 yds., side of sq. = 49·193 yds.

10 (19) 80 sq. ft. (20) 465·812, 408·259, 70

11 (21) 168077·17 sq. miles. (22) 1258·354 ft.

12 (23) 24, 452·39. (24) 4·6366, 16·884. (25) 19·75
(26) 27·3. (27) 36·53, 15·10.

ANSWERS TO QUESTIONS IN PROB. 5.

Sec.

1 (1) 60. (2) 54·75.

2 (3) 72·291. (4) 194·8.

3 (5) 85. (6) 5567·66.

4 (7) 52. (8) 55·5. (9) 156452·8.

5 (10) 59. (11) 874. (12) 4472·7.

6 (13) 2026·6. (14) 197. (15) 117412·62.

7 (16) 259·3. (17) 3464390·94.

8 (18) 5136·45. (19) 100.

9 (20) 4378·33. (21) 776. (22) 394¾.

ANSWERS TO QUESTIONS IN PROB. 6.

Sec.

	Feet.	Feet.	Feet.
1	(1) $X=$ 5·487,	$Y=$2·615.	(2) $X=$ 7·388, $Y=$
2	(3) $X=$ 3·8,	$Y=$4·796.	(4) $X=$ 7·803, $Y=$
3	(5) $X=$ 9·01,	$Y=$5·62.	(6) $X=$ 8·32, $Y=$
4	(7) $X=$ 6·234,	$Y=$2·887.	(8) $X=$ 5·695, $Y=$
5	(9) $X=$31·414,	$Y=$7·98.	(10) $X=$42·434, $Y=$1
6	(11) $X=$ 9·076,	$Y=$5·535.	(12) $X=$11·269, $Y=$
7	(13) $X=$ 9·66,	$Y=$2·966.	(14) $X=$48·86, $Y=$
	(15) $X=$45·34,	$Y=$5·81.	

ANSWERS TO QUESTIONS IN PROB. 7.

Sec.		
1	(1) 18·849 sq. ft.	(2) 34·557 sq. ft.
	(3) 64·256 sq. ft.	(4) 32·288 sq. ft.
2	(5) 36·221 sq. ft.	(6) 180·118 sq. ft.
	(7) 29·06 sq. ft.	(8) 138·754 sq. ft.
3	(9) 53.489 sq. ft.	(10) 17·819 sq. ft.
	(11) 119·463 sq. ft.	(12) 145·655 sq. ft.
4	(13) 29·698 sq. ft.	(14) 27·489 sq. ft.
	(15) 129·592 sq. ft.	(16) 56·942 sq. ft.
5	(17) 73·124 sq. ft.	(18) 13·614 sq. ft.
	(19) 20·35 sq. ft.	(20) 4·45 sq. ft.
6	(21) 145·119 sq. ft.	(22) 91·449 sq. ft.
	(23) 119·7 sq. ft.	(24) 195·973 sq. ft.
7	(25) 342·258 sq. ft.	(26) 171·516 sq. ft.
	(27) 2·7489 sq. ft.	(28) ·9817 sq. ft.
8	(29) 1848·4 tons.	(30) 1241·2 tons.
9	(31) 1995·6 tons.	(32) 620·8 tons.
10	(33) 2933·7 tons.	(34) 565·9 tons.
11	(35) 16·93 tons.	(36) 14·21 tons.
12	(37) 5·657 diam.	(38) 5·196, 7·348 diam.

ANSWERS TO QUESTIONS IN PROB 8.

Sec.	
1	(1) 107·4, 118·46.
2	(2) 607·4, 3870·4, 1951·1.
3	(3) 803·4, 1178, 2429.

ANSWERS TO EXAMPLES IN PROB. 9.

Sec.			
1	(1) 7·6626, 7·986, 189·6.	(2) 3·527, 4·359. 45·19.	
2	(3) 7.917, 8·787, 211·28.	(4) 10·68, 12·33, 395·19.	
3	(5) 12·134, 12·91, 482·35.	(6) 13·528, 14·006, 588·5.	
4	(7) 5 a. 2 r. 23½ p.	(8) 4 a. 2 r. 18½ perches.	
5	(9) 53·039 yds.	(10) 4·5509 yds.	(11) 12·54 yds.
6	(12) 1·376.	(13) ·002046.	(14) 94·6.

ANSWERS TO EXAMPLES IN PROB. 10.

Sec.			
	° ′ sq. ft.	° ′ sq. ft.	° ′ sq. ft.
1	(1) 19 35, 91·41.	(2) 17 5, 77·24.	(3) 21 40, 89·74.
2	(4) 21 20, 74·81.	(5) 18 55, 56·1.	(6) 24 10, 8·88.
3	(7) 21 40, 50·48.	(8) 25 30, 8·08.	(9) 16 25, 62·3.
4	(10) 21 40, 89·743.	(11) 19 25, 8·17.	(12) 23 25, 56·64.
5	(13) 18 30, 46·98.	(14) 11 10, 25·3.	(15) 19 15, 22·83.
6	(16) 16 15, 24·15.	(17) 19 40, 21·58.	(18) 17 40, 26·24.
7	(19) 22 25, 7·4.	(20) 20 35, 67·08.	(21) 21 15, 63·29.

ANSWERS TO EXAMPLES IN PROB. 11.

(1) 17·72 ft. (2) 15·3. (3) 10·23. (4) 9·99. (5) 14·07.

MENSURATION OF SOLIDS.

ANSWERS TO EXAMPLES IN PROB. 1.

Sec.

1 (1) 140 c. ft. (2) 228·75 c. ft.
(3) 4·0115 ft.

2 (4) $1808\frac{1}{28}$ tons. (5) 84 c. ft., 6·15 tons.
(6) Weight = 396·6 lbs., hold 267·49 gallons.

3 (7) 2 t. 5 cwt. 3 qr. 23 lbs. 3 oz. (8) 2 t. 0 c. 3 q. 10 lb. 15·9 oz.
(9) 30 t. 9 cwt. 5 lbs. 13 oz.

4 (10) 3 t. 17 cwt. 3 qr. 0 lb. 7 oz. (11) 146·827 cub. ft.
(12) 1 t. 18 cwt. 3 qr. 1 lb. 4 oz.

5 *(13) 96·9233* c. ft. (14) 626·165 lbs.
(15) 171·027 c. ft.

Sec.

6 (16) 776·367 lbs. lost (17) 18 cwt. 17 lbs. 5·4 oz.
(18) 2 t. 10 cwt. 17 lbs. 15·4 oz.

7 (19) 1 t. 4 cwt. 22 lbs. 15 oz. (20) 10*l.* 15*s.* 3*d.*
(21) 21*l.* 7*s.* 1¾*d.*

8 (22) 4 t. 0 cwt. 2 qr. 9 lbs. 7 oz. (23) 1246·22 gallons.
(24) 68·3313 c. ft. (25) 2 t. 4 cwt. 2 qr. 17 lbs. 8 oz.

9 (26) 10469. (27) 1955.
(28) 2485.

ANSWERS TO EXAMPLES IN PROB. 2.

Sec.

1 (1) 66·8. (2) 44·72.

2 (3) 17·65. (4) 7·0555 c. ft., 1*l.* 4*s.* 8¼*d.*

3 (5) 29·39 c. ft., 2·151 tons. (6) 18·25, 858·84 lbs.

4 (7) 58·663. (8) $54\frac{5}{27}$.
(9) 148·38. (10) 26·615.

5 (11) $\frac{12}{24}$ c. ft. (12) 2½ c. ft.
(13) 4·2937 c. ft., 250·623 lbs.

ANSWERS TO EXAMPLES IN PROB. 3.

Sec.

1 (1) 35·343 c. ft. (2) 139·162 c. ft.
(3) 251·016 c. ft. (4) 119·036 c. ft.

2 (5) 98·08 c. ft. (6) 308·961 c. ft.
(7) 638·2 lbs.

3 (8) 3·437 tons. (9) 23·98 c. ft.
(10) 1·7927 tons.

4 (11) 232·55 c. ft. (12) 541·2 lbs.
(13) 183·2 c. ft.

5 (14) 100·54 c. ft. (15) 610·73 c. ft.

6 (16) 1402·1 c. ft. (17) 659 lbs.

7 *(18) 3413·7l.* (19) 2356*l.* 4*s.*

ANSWERS TO EXAMPLES IN PROB. 4.

Sec.

1 (1) 3053·6 c. ft., 1017·9 sq. ft.

(2) 7238·24 c. ft., 1809·56 sq. ft.

(3) 8181·25 c. ft., 1963·5 sq. ft.

2 (4) 6795·21 c. ft., 1734·9 sq. ft.

(5) 47·7 c. ft., 63·6 sq. ft.

3 (6) 350·8 c. ft., 240·5 sq. ft., 11 t. 18 cwt. 3 qr.

4 (7) 665·7 c. ft., 368·69 sq. ft., 13 t. 19 cwt. 2 qr. 23 lbs.

5 (8) 45·1 c. ft., 61·3 sq. ft., 10 t. 11 cwt. 1¾ qr.

6 (9) 222·5 sq. ft., 312·2 c. ft., 143·9 c. ft. displacement.

7 (10) 1 t. 7 cwt. 3 qr. 16 lbs. 6 oz., 108·45 c. ft., 109·99 sq. ft.

8 (11) 1898·6 sq. ft., 7778·897 c. ft., 202 t. 14 cwt., 7264·9 c. ft. di

9 (12) 3·1416 sq. ft. 12·566 sq. ft. 28·274 sq. ft.
·5236 c. ft. 4·1888 c. ft. 14·1372 c. ft.

50·265 sq. ft. 78·54 sq. ft. 113·097 sq. ft.
33·51 c. ft. 65·45 c. ft. 113·097 c. ft.

153·938 sq. ft. 201·062 sq. ft. 254·47 sq. ft.
179·594 c. ft. 268·08 c. ft. 381·7 c. ft.

314·16 sq. ft.
523·6 c. ft.

10 (13) 3·579 ft., 1400·9 lbs., 40·22 sq. ft.

11 (14) 4·09 ft., 2745 lbs., 52·7 sq. ft.

12 (15) 5·64 ft., 99·98 sq. ft., 70·78 c. ft.

13 (16) 5·43 ft., 92·7 sq. ft., 37096 lbs.

14 (17) 3·94 ft., 48·7 sq. ft., 29·8 c. ft.

15 (18) 4·31 ft., 58·4 sq. ft., 19·36 c. ft.

16 (19) 3·33 yds., 19·47 c. yds., 5·17 yds., 72·39 c. yd

17 (20) 5·47 ft., 85·69 c. yds. 4·184 ft., 38·35 c. yd

18 (21) 4·51 ft., 48.1¼ c. yds., 5·17 ft., 72·39 c. yd

19 (22) 7·82 in., 6·2 in.

20 (23) 4·47 in., 5·12 in., 5·64 in.

21 (24) 6·79 in., 7·98 in., 8·56 in.

Sec.

22 (25) Diam. $= 1\frac{1}{2}$, vol. $= \frac{9\pi}{16}$.

(26) Diam. $= \sqrt{6}$, vol. $= \pi\sqrt{6}$.

(27) Diam. $= \frac{3+\sqrt{105}}{4}$, vol. $= \frac{\pi}{32}(81+11\sqrt{105})$.

23 (28) Diam. $= 6$, vol. $= 36\pi$.

(29) Diam. $= \sqrt{3}(\sqrt{3}+\sqrt{5})$, sur. $= 6(4+\sqrt{15})\pi$,
vol. $= (27+7\sqrt{15})\pi$.

24 (30) 202·63 c. in., 51·7 lbs. (31) ·1554 c. ft., 74·79 lbs.
(32) ·7196 c. ft., 42·005 lbs. (33) ·6463 c. ft., 30·415 lbs.

25 (34) 7·4 in. (35) ·7445 c. ft., 400·48 lbs.

26 (36) 24·549 galls., 4*l.* 15*s.* $0\frac{3}{4}$*d.* (37) 10·805 in.

27 (38) 34·86 in. (39) 12·4 in.
(40) 172·05 lbs., 297·3 lbs., 407·83 lbs.

ANSWERS TO EXAMPLES IN PROB. 5.

Sec.

1 (1) 35·343 c. ft., 52·36 c. ft., 268·0832 c. ft.
(2) 45·815 c. yds., 30·066 c. yds., 836·29 c. ft.

2 (3) 4419·708 c. ft., 1893·858 c. ft., 1387·1 c. ft.

3 (4) 10·39 c. ft.
(5) 24 (6) 19·476.

4 (7) 348·23 lbs. (8) 6 t. 1 cwt. 3 qr.

5 (9) 193·16 sq. ft., 1383·52 sq. ft., 164·268 c. ft. 1860·77 c. ft.

6 (10) 79·44 tons, 16·17 tons.

7 (11) 1827·02 c. ft. displacement, 1956·3 c. ft.

8 (12) 4·363 c. ft., 1 t. 7 cwt. 3·8 qr.
(13) 6629.57 c. ft., 3055·9 c. ft.

9 (14) 113·098 sq. ft., 1 t. 6 cwt. 22 lbs., 50·265 c. ft.
(15) 198·13 c. ft., 301·59 sq. ft., 301·59 c. ft.

10 *(16) 12·61 c. ft., 155·8 c. ft., 166·3 c. ft.*

ANSWERS TO EXAMPLES IN PROB. 8.

Sec.		
1	(1) 1526·82 c. ft.	(2) 4515·52 c. ft.
	(3) 4771·3 c. ft.	
2	(4) 769·69 c. ft.	(5) 1910·09 c. ft.
	(6) 2327·92 c. ft.	
3	(7) 35·6 c. ft.	(8) 372·28 c. ft.
	(9) 84·823 c. ft.	(10) 1072·07 c. ft.
4	(11) 1488·33 c. ft.	(12) 503·94 c. ft.
	(13) 183·78 c. ft.	(14) 12·37 c. ft.
5	(15) 3351·04 c. ft.	(16) 10786·6 c. ft.
	(17) 35·78 c. ft.	(18) 1470·27 c. ft.
6	(19) 84·82 c. ft.	(20) 45·23 c. ft.
7	(21) 508·94, 1055·57, 1107·41, 32·076 tons, 94·866 ton 100·24 tons.	
8	(22) 43·982, 75·398, 245·04, 21·2, 2078·2 lbs., 2·21 ton 13·262 tons, 722 lbs.	
9	(23) 879·64, 42·411, 75·39, 114 tons, 1·2181 tons, 2·88 tons.	
10	(24) 14 t. 4 cwt. 3 qr.	(25) 748.45 lbs.
11	(26) 938·29 c. ft., 958·19 sq. ft.,	502·656 sq. ft., 568·1 c. ft., 2994·47 c. ft., 279·6 sq. ft.
12	(27) 701·6 c. ft., 785·4 sq. ft.,	333·01 sq. ft., 4599·3 c. ft. 2770·89 c. ft. 1068·1 sq. ft.
13	(28) 183·78 c. ft., 289·81 sq. ft., 3396·72 c. ft.,	141·37 sq. ft., 643·24 c. ft., 1653·5 c. ft., 547·42 sq. ft., 871·01 sq. ft.
14	(29) 6352·3 c. ft., 2277·6 sq. ft..	1360·3 sq. ft., 7612·62 c. ft. 1293·3 c. ft., 468·1 sq. ft.
15	(30) 7068·6 c. ft., 911 sq. ft.,	1413·7 sq. ft., 8621·2 c. ft., 3152·59 c. ft., 870·2 sq. ft.

ANSWERS TO EXAMPLES IN PROB. 9.

Sec.		
1	(1) 2513·2.	(2) 70·68.
	(3) 502·65.	(4) 167·55.
2	(5) 1832·6.	(6) 1154·5.
	(7) 636.17.	(8) 259,791,592,621.
3	(9) 393·6.	(10) 12274·1.
	(11) 718·38.	(12) 2238·02.
4	(13) 2109.	(14) 1717·2.
	(15) 60·27.	(16) 183·73.

ANSWERS TO EXAMPLES IN PROB. 10.

Sec.		
1	(1) 421·9 c. in., 562·56 sq. in.	
	(2) Vol. = 1332·4 c. in., 888·2 sq. in.	
2	(3) 17·9 lbs.	
	(4) Vol. = 4105·8 c. in., surf. = 2052·9 sq. in.	
3	(5) 16·603 galls.	(6) 35·29.
4	(7) 1134 c. ft.	(8) 66·3 galls.

ANSWERS TO QUESTIONS IN SURVEYING.

Sec.		
1	(1) 4 a. 2 r. 19·8 p.	(2) 3 a. 3 r. 11 p.
2	(3) 48 a. 0 r. 11¼ p.	(4) 34 a. 0 r. 13½ p.
3	(5) 25 a. 2 r. 24 p.	(6) 13 a. 3 r. 23¾ p.
4	(7) 9 a. 1 r. 6 p.	(8) 33 a. 1 r. 16 p.
5	(9) 14 a. 2 r. 10·19 p.	(10) 22 a. 1 r. 9·3 p.
6	(11) 10 a. 1 r. 17·4 p.	(12) 9 a. 2 r. 38·27 p.
	(13) 10 a. 0 r. 26¼ p.	(14) 5 a. 1 r. 19·4 p.

ANSWERS TO QUESTION IN THE APPLICATION OF ALGEBRA TO MENSURATION.

Section 1.

(1) The solid of the prismoid,

$$\frac{12\,V - B\,L\,(2\,h + h')}{L\,(2\,h' + h)}\;;\quad \frac{12\,V - h\,L\,(2\,B + B')}{L\,(2\,B' + B)}\;;$$

the volume of the wedge.

(2) The volume of the segment of a sphere whose diameter is D, and height of segment h.

$$\frac{2\,V}{\pi\,h^2} + \frac{2\,h}{3}\;.$$

(3) The volume of the segment of a sphere whose diameter of base is (b), and height of segment h.

$$\sqrt{\frac{8\,V}{\pi\,h} - \frac{4\,h^2}{3}}\;;\;\frac{2\,V}{\pi\,h^2} + \frac{2\,h}{3}\;.$$

(4) The surface of the segment of a sphere whose height is h, and the diameter of the base b.

$$2\sqrt{\frac{s}{\pi} - h^2}\;;\;\frac{s}{\pi\,h}\;.$$

(5) The volume of a concentric cylinder whose internal and external diameters are d and D, and length L.

$$\sqrt{\frac{4\,V}{\pi\,L} + d^2}\;;\;\sqrt{D^2 - \frac{4\,V}{\pi\,L}}\;.$$

Section 2.

(6) The volume of a hollow sphere whose internal and external diameters are d and D.

$$\left(\frac{6\,V}{\pi} + d^3\right)^{\frac{1}{3}}\;.$$

(7) The difference between the outer and inner surfaces of a hollow sphere whose diameters are D and d.

$$\sqrt{\frac{s}{\pi} + d^2}\;.$$

(8) The volume of a cone whose diameter is D, and height h.

$$\sqrt{\frac{12\,V}{\pi\,h}}\;.$$

Section 2.—*(continued.)*

(9) The volume of a frustum of a cone, whose diameters are D and d, and length h.

$$\pm \sqrt{\frac{12\,V}{\pi\,h} - \frac{3\,d^2}{4}} - \frac{d}{2}.$$

(10) 1. Circumference of a circle whose diameter is D. 2. Area of a circle whose diameter is D. 3. Area of an ellipse whose diameters are D and d. 4. Area of a parabola whose ordinate is y, and abscissa x.

Section 3.

(11) $4\,\pi\,a^2$ (12) $\frac{4\,(a-\pi)^2}{\pi}$ (14) $\frac{4^{\frac{1}{3}}\,(a\,\pi+b)^{\frac{2}{3}}}{\pi^{\frac{1}{3}}}$

Section 4.

(15) $\pi\,(3 \pm 2\,\sqrt{2})$ (17) $4\,\pi + \frac{1}{4\,\pi} + 2$

(18) $\frac{1 + 8\,\pi \pm \sqrt{1 + 16\,\pi}}{8\,\pi}$

Section 5.

(19) $2\sqrt{\frac{a}{\pi}}$ (20) $4\,\pi^3$ (21) $a + 2\,\pi \pm 2\,\sqrt{\pi^2 + a\,\pi}$

Section 6.

(22) $\sqrt{\frac{2\,a}{n}}$ (24) $\frac{(\pi+1)^2}{\pi}$ (25) $12\,\sqrt{3}$

Section 7.

(26) $\frac{2\,(\pi^2+1)^2}{\pi}$ (27) $2\,r^3$ (28) $\frac{3\,d + 2 \pm \sqrt{d^2+12\,d+4}}{2}$

Section 8.

(29) 24 (30) 3 & 7 (31) $\frac{2\,(n+1)^2}{n}$

(32) $\frac{\sqrt{1+16\,n}-1}{4}$

Section 9.

(33) $2\pi\sqrt{\dfrac{a}{\pi(\pi^2-1)}}$ (34) $\dfrac{4(\pi-1)}{\pi(\pi+1)}$ (35) $\dfrac{3\pi\delta^2}{2(\pi\delta-2)}$

(36) $\dfrac{4r^4}{\pi(r^2-1)}$

Section 10.

(37) $2\pi^2 L^2$ (38) $\dfrac{4\pi r^4 L}{r^2-1}$ (39) $\dfrac{3\delta^2}{2(\delta-2\pi L)}$

Section 11.

(40) $\dfrac{2\sqrt{3}}{r^2\sqrt{\pi}}$ (41) $\dfrac{3(1+\delta)\pm\sqrt{\delta^2+18\delta+9}}{2}$

(42) $12\pi\delta^3$

Section 12.

(43) $\dfrac{9}{2}\left\{4\pi+2+(4\pi+\tfrac{2}{3})\sqrt{1+\dfrac{2}{3\pi}}\right\}$

$18\left(\pi+\dfrac{1}{3}+\pi\sqrt{1+\dfrac{2}{3\pi}}\right)$

(44) $(6\pi)^{\frac{1}{2}}$; 6 (45) $D\left(\dfrac{n-1}{n}\right)^{\frac{1}{2}}$

Section 13.

(46) $\dfrac{9}{2}\left\{1+\pi+(\pi+\tfrac{1}{3})\sqrt{1+\dfrac{4}{3\pi}}\right\}$; $\dfrac{9}{2}\left\{\pi+\tfrac{2}{3}+\sqrt{\pi^2+\dfrac{4\pi}{3}}\right\}$

(47) $\dfrac{2\pi h^3}{3(h-2)}$

Section 14.

(49) $2r^2\sqrt{\dfrac{6r}{\pi(3+4r^2)}}$

(50) $\dfrac{2}{5}\left\{\pm\dfrac{1}{\pi}\sqrt{1+6\pi\delta-\pi^2\delta^2}+3\delta+\dfrac{1}{\pi}\right\}$

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